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Less is more: Testing financial integration using identification-robust asset pricing models ☆

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ABSTRACT

We revisit financial market integration and study the impact of multiple risk factors and model specification on inference. Our tests exploit a method correct in finite sample that jointly assesses coefficient significance and detects identification problems. Results on four countries show that multiple sources of risk in international asset pricing models lead to lack of identification and spurious inference. We find that domestic factor models are well identified which is not the case for global and international models. Nonetheless, domestic models do not provide a base for testing financial market integration. Given that constraint, the best-identified international model includes few factors and reveals that financial integration varies over time and across countries.

1. Introduction

Testing for financial market integration implies that we can rely on the underlying model. Therefore specification of international asset pricing models has important implications for our understanding of the geographic sources of systematic risk as well as potential strategies for risk diversification. Of crucial importance in developing these models is our ability to determine risk factors and to detect model misspecification. In this context, an important question is whether the explanatory power of risk factors is present locally or globally. Some studies argue that only local, country-specific factors, constructed from firm characteristics, matter for global stock returns (Griffin, 2002). Zhang (2006) finds that country risk factors are not priced over global risk factors. Others perceive a globally integrated market, and advocate for models that incorporate both local and foreign factors, built from firm characteristics (Bekeart et al., 2009; Karolyi and Wu, 2012). This literature often

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relies on the well-known Gibbons et al. (GRS) (1989) statistic (e.g. Griffin, 2002; Fama and French, 2012). In this paper, we go beyond this issue and argue that while increasing the number of risk factors appears to improve model fit, it can also lead to identification problems and spurious test results and importantly affect our conclusions on financial market integration. We exploit data from four countries between 1990 and 2011 to study how international finance models and integration tests fare when estimated with an identification-robust and finite-sample correct method.

1.1. What is a well-identified model

Strictly speaking, an identified model is one from which it is possible to draw inferences from the probability distribution of the observed variables to an underlying theoretical structure (Dufour and Hsiao, 2008).Therefore, a well-identified model in the statistical sense is a model where the data yields reliable information about the estimated parameters. In addition to the "pitfalls" highlighted by Lewellen et al. (2010), two related features can undermine identification: (i) redundant factors *i.e.* betas or linear combination of betas that are close to zero, and (ii) clustering, *i.e.* betas that do not differ much across test assets, for example are close to one.

1.2. Why the problem of identification is important in international asset pricing

From an asset pricing perspective, identification is a serious problem since, traditional[4] Wald-type confidence intervals and tests will seriously under-cover and over-reject (Dufour, 1997; Beaulieu et al., 2013; Kleibergen and Zhan, 2015; Kan et al., 2013; Khalaf and Schaller, 2016; Gospodinov et al., 2013). Wald or Student-type confidence intervals are intervals of the form: {estimate ± (standard error) × critical point}. When identification is not granted, estimates are biased and standard errors severely understate estimation uncertainty, even with large samples. So in this case, as opposed to the very large standard errors one would expect,[5] traditional inference produces unduly small standard errors which in conjunction with bias, produces intervals that are tightly concentrated on false values. In addition to spurious inference, this implies that identification problems are hard if not impossible to detect with estimated intervals, in view of their tightness.

This paper provides a solution to detect such problems in international contexts where the proliferation of risk factors exacerbates the issue. We exploit an inverted test, developed by Beaulieu et al. (2013). This is the only method available that is both robust to identification problems and correct in finite sample. The proposed confidence *sets* (as opposed to Wald-type confidence *intervals*) cover the true parameter value according to the desired level, whether identification holds or not. This being said, our objective is not to prove lack-of-identification in an absolute sense, but report diffuse sets as important flag for identification.

1.3. Contributions to the literature

Our first contribution is to document the performance (in terms of model fit and identification) of several popular international asset pricing models. The literature recognizes identification problems in asset pricing resulting from a large number of factors, non-informative factors or weakly varying betas (e.g. Gospodinov et al., 2014; Kan et al., 2013; Beaulieu et al., 2013; Kleibergen and Zhan, 2015; Harvey et al., 2015). Empirically, the international context has not yet been analyzed. Our results suggest that relying solely on the GRS statistic to measure an international asset pricing model fit can be misleading when it comes to model assessment. Our empirical analysis reveals cases in which these models are not identified, yet their GRS statistic is not statistically significant at usual levels. While the GRS statistic remains an important tool to analyze model performance, non-rejections should be interpreted with caution in the presence of many factors. We show that the performance of international asset pricing models can be adversely affected by a large number of factors – which is likely to be the case in international finance – given the presence of multiple factors emanating from multiple sources of risk.

Our second contribution is to provide a joint analysis of financial integration along with the assessment of the underlying models studied. From a substantive perspective, we revisit financial market integration when the factor base is imprecise as to the source of risk (domestic or global) and when theory provides no guidance for factor selection (market or firm-specific based). We find that the domestic risk component is often priced in our set of countries. We also find that financial integration is time varying with no evidence of a trend towards integration. Finally, even for a well-identified model, we note changes of decision between our results – which are correct in finite samples – and those based on asymptotic Wald tests which, in view of the existing literature and the above discussion (e.g. Dufour and Dagenais, 1992 and Beaulieu et al., 2013), documents the Wald test limitations in terms of invariance to portfolio parameterization and underscore the importance of our alternate approach.

[4] The problems we refer to here cover all usual estimates and standard errors of the OLS, GLS, MLE and GMM form, and of the Fama-McBeth type even when corrected for measurement errors.

[5] As would occur, for example, with quasi-colinear regression. Non-linearity and resulting lack of invariance to reparametrization "masks" this phenomenon (Dufour and Dagenais, 1992; Dufour, 1997).

2. Framework and notation

2.1. General framework

Shanken and Zhou (2007) study restrictions on the intercepts of traditional equilibrium asset pricing models and provide an explicit passage from the two-pass estimation of cross-sections proposed in Fama and MacBeth (1973) to a one-pass estimation method. We estimate international asset pricing models in a multivariate regression (MR) framework building upon their approach.

Consider the following general empirical asset pricing model:

$$r_{it} = a_i + \sum_{j=1}^{q} \beta_i^j \tilde{r}_{jt} + u_{it}, \quad i = 1, \ldots, n \text{ and } t = 1, \ldots T \tag{1}$$

where r_{it} are the excess returns (over risk free rate or inflation) excess return on portfolio i at time t, $\tilde{r}_{jt}$ are returns on q benchmark factors, β_i^j are the sensitivities associated with each risk factor j for portfolio i and u_{it} is a random disturbance. We use different constraints on the intercept (a_i) and on the nature of the data used in r_{it} and $\tilde{r}_{jt}$ to estimate and compare the performance of several well-known international asset pricing models used in the international finance literature. In each case, the international asset pricing models include: (1) a closed economy in the presence of domestic factors; (2) an integrated economy using global factors only and (3) an international setting which includes both global and domestic risk factors that allows for a test of financial integration.

In all specifications, we evaluate the model's fit and identification, as well as relevant hypotheses of integration. Testing for the joint significance of the unconstrained intercept, a_i, is a well-known test for model assessment (GRS, 1989). We implement this test to investigate the quality of each asset pricing model such as presented in Mittoo (1992), Griffin (2002) and Fama and French (2012). Furthermore, we investigate the joint significance of each risk factor introduced into a given model. Since redundant factors have important implications in terms of asset pricing and identification, as underscored by Kan and Zhang (1999), Beaulieu et al. (2013), Khalaf and Schaller (2016), Gospodinov et al. (2013), we test for the joint significance of the β_i^j in all specifications. Thus, in all models, the following hypotheses are tested:

$$H_0 : a_i = 0, \ \forall i \tag{2}$$

$$H_{0j} : \ \textit{for each risk factor } j : \ \beta_i^j = 0, \ \forall i. \tag{3}$$

2.2. International asset pricing models

We first describe the constraints that define our asset pricing models as applied to Eq. (1). Second, we present our general testing methodology within the context of estimating Eq. (1).

2.2.1. Empirical factor models

In addition to the market factor, we follow Fama and French (1992, 1993) and Carhart (1997) and introduce benchmarks in the form of firm-specific factors: size (SMB), book-to-market (HML) and momentum (WML). We also include a foreign exchange risk factor (forex) (Zhang, 2006). Each factor is added one at a time to estimate two-factor models (market and size as well as market and forex), the Fama and French original model (market, size and book-to-market), the four factor model (market, size, book-to-market and momentum), another model with four factors including forex (market, size, book-to-market and forex) and a five factor model with all risk factors (market, size, book-to-market, momentum and forex). In what follows, we present the five factor model, since it nests the other models which can easily be recovered from the general case by constraining the coefficient of the omitted factor to zero. Note that forex is included only in global and international models.[6]

2.2.1.1. Domestic factor models. When markets are segmented, the appropriate risk factors represent domestic systematic risk. Shanken and Zhou (2007) show that in multifactor models – the four-factor model $\tilde{r}_{jt} = [r_{MKTDt}\ r_{SMBDt}\ r_{HMLDt}\ r_{WMLDt}]$ in our case – market equilibrium implies the following non-linear constraint on the intercept:

$$a_i = -\delta^{MKTD}\beta_i^{MKTD} - \delta^{SMBD}\beta_i^{SMBD} - \delta^{HMLD}\beta_i^{HMLD} - \delta^{WMLD}\beta_i^{WMLD}. \tag{4}$$

We define δ^{MKTD}, δ^{SMBD}, δ^{HMLD}, δ^{WMLD} as the model "*deltas*". Restriction (4) which introduces these parameters into the econometric model is standard in asset pricing. Indeed, plugging (4) into (1) and re-arranging terms, (4) simply implies zero intercepts, in the regression of excess returns on a constant and factors, each in excess of its corresponding "*delta*". Typically, the *deltas* are unknown, in which case (4) introduces non-linearity in parameters. Furthermore, the *deltas* easily map into the so-called *factor risk premiums* which typically parametrize cross-sectional interpretations of (1) and (4). As is well known

[6] We also estimated domestic models augmented with the forex risk factor. The results were not importantly affected by the inclusion of the forex factor and are omitted for brevity.

(see e.g. Shanken and Zhou, 2007, and references therein), the *cross-sectional risk premiums* equal the *deltas* minus the factor means. All of the above implies the following: (i) inference on the *deltas* validates the empirical asset pricing model from both time series and cross-sectional interpretations; (ii) inference on factor pricing, that is, on whether a factor cross-sectional risk premium is zero, implies checking whether its corresponding *delta* equals its *mean*:

$$H_{0\delta^j} : \delta^j = \mu_j \tag{5}$$

where μ_j refers to the mean in question.

It is important to pin-point the potential identification problem in (4). Observe that if for example β_i^{SMBD} clusters around zero for all portfolios, then δ^{SMBD} simply disappears from the model and thus should be non-recoverable using any statistical objective function. This invalidates usual inference methods. In contrast, our method works even under such conditions. We emphasize that we assess pricing via the *deltas*. We first derive finite sample simultaneous (in the sense of joint coverage) confidence intervals for these *deltas*, which inform us on model identification and fit. We next check whether each of these intervals, in turn, covers the corresponding factor empirical mean, which will inform us on factor pricing. The model specification and factor pricing checks are thus performed in a single statistical step, to which only one type-I error is associated.

2.2.1.2. Global factor models. When markets are integrated, the model contains returns on global risk factors. In the five-factor model with $\tilde{r}_{jt} = [r_{MKTGt}\ r_{SMBGt}\ r_{HMLGt}\ r_{WMLG}\ r_{forext}]$, the constraint on the intercept is:

$$a_i = -\delta^{MKTG}\beta_i^{MKTG} - \delta^{SMBG}\beta_i^{SMBG} - \delta^{HMLG}\beta_i^{HMLG} - \delta^{WMLG}\beta_i^{WMLG} - \delta^{forex}\beta_i^{forex}. \tag{6}$$

The above defined inference on factor pricing via each of the *deltas* presented in Eq. (5) is also investigated in this model, as explained above.

2.2.1.3. International factor models. When markets are partially segmented or partially integrated, the model is similar to the international model presented in Griffin (2002). Jorion and Schwartz (1986) and Mittoo (1992) note that the domestic and global sources of risk are highly correlated in which case tests of financial integration are not informative. Thus, the purely domestic risk factor *j*, $r_{JD\perp t}$, needs to be purged of the effect of the global risk factor *j* (r_{JGt}) via a preliminary regression. The residual of the ordinary least squares (OLS) regression of the domestic risk factor on a constant and the global risk factor is used in place of the domestic market return:

$$r_{JD\perp t} = r_{JDt} - \widehat{constant} - \widehat{beta}_1 r_{JGt} \tag{7}$$

Here, both global and domestic sources of risk are priced and the domestic risk factors are orthogonal to their global risk factor equivalent by construction. The residuals also have a mean of zero by construction. The constraint on the intercept is:

$$\begin{aligned} a_i = & -\delta^{MKTG}\beta_i^{MKTG} - \delta^{MKTD\perp}\beta_i^{MKTD\perp} - \delta^{SMBG}\beta_i^{SMBG} - \delta^{SMBD\perp}\beta_i^{SMBD\perp} - \delta^{HMLG}\beta_i^{HMLG} - \delta^{HMLD\perp}\beta_i^{HMLD\perp} - \delta^{WMLG}\beta_i^{WMLG} \\ & - \delta^{WMLD\perp}\beta_i^{WMLD\perp} - \delta^{forex}\beta_i^{forex}. \end{aligned} \tag{8}$$

As a special case of Eq. (5), financial integration is tested with

$$H_{0j\perp} : \delta^{j\perp} = \mu_j^{\perp} = 0. \tag{9}$$

which, when the null cannot be rejected, implies that markets are integrated with respect to risk factor *j* (if it is priced) since only the global component of this factor is priced in equilibrium. If no domestic risk factors are priced in equilibrium, the model reverts to the global model and markets are integrated with respect to every risk factor.

2.2.2. Black CAPM

Given that the models above include many risk factors and lead to identification problems, we consider alternative models. Our objective is also to present a well-identified model to implement financial integration tests. An important extension of the CAPM consists in allowing for restrictions on borrowing (Black, 1972). In a more general model, portfolio mean–variance efficiency is defined using expected returns in excess of the zero-beta rate (γ).

2.2.2.1. Black ICAPM. More importantly for our purpose, Mittoo (1992) presents a version of the model which provides a test for financial integration. Similarly to the international empirical factor model, the purely domestic market index, $r_{MKTD\perp t}$, needs to be purged of the effect of the global market index via a preliminary regression presented in Eq. (7). The $T \times 2$ matrix of risk factors is: $\tilde{r}_{jt} = [r_{MKTGt}, r_{MKTD\perp t}]$. In this ICAPM model, the constraint on the intercept (Mittoo, 1992) is:

$$a_i = \gamma(1 - \beta_i^{MKTG}) - \delta^{MKTD\perp}\beta_i^{MKTD\perp}, \tag{10}$$

where $\delta^{MKTD\perp}$ is the risk premium associated with the purely domestic component of market return. Note that if $\delta^{MKTD\perp} = 0$, the model reverts to the global model presented below. In the spirit of Mittoo (1992) and Jorion and Schwartz (1986) the international model provides an intuitive test for financial integration. Financial markets are integrated when the purely domestic component of systematic risk is not priced in equilibrium (Campbell et al., 1997):

$$H_{0INT}: \delta^{MKTD\perp} = \mu_{MKTD\perp} = 0 \tag{11}$$

2.2.2.2. Domestic and global Black CAPM. If the international version of the model is inappropriate because markets are perfectly segmented or integrated, the model includes one benchmark, namely returns on the domestic or global market portfolios, $r_{MKTDt}r_{MKTGt}$. The intercept on the constrained version of the model takes the following form (MacKinlay, 1987) for the domestic version:

$$a_i = \gamma(1 - \beta_i^{MKTD}). \tag{12}$$

The global version is equivalently a constrained version of the international model where $\delta^{MKTD\perp} = 0$ and is:

$$a_i = \gamma(1 - \beta_i^{MKTG}). \tag{13}$$

Note that these non-linear constraints in all of the above models present a potential identification issue. For example, if the β_i^{MKTD} are close to one (which is often the case in finance), the resulting parameter multiplying γ is $1 - \beta_i^{MKTD} \cong 0$. Any value of gamma when multiplied by an expression that is close to zero for all portfolios cannot be recovered from available data (Dufour, 1997 and Beaulieu et al., 2013). It is therefore an empirical question whether these models are better identified than factor models described in Section 2.2.1.

2.3. Testing methodology

We now present our general testing methodology which requires two different Hotelling statistics. The first one is presented in Beaulieu et al. (2010) and used to test for the joint significance of the intercept and factors as presented in Eqs. (2) and (3). The second one is an extension of the Hotelling statistic in Beaulieu et al. (2013) to an international setting. We use it to test for the significance of structural parameters and assess model specification as stated in the hypotheses in Eqs. (5), (9), and (11). In what follows, we develop the test statistic for the most general case.[7]

Let us first briefly describe our method to facilitate its formal definition. The parameters (*deltas* in our notation see Eqs. (4), (6) and (8)) are linked one-to-one to the factor premiums which typically parametrize cross-sectional regressions. The one-to-one mapping is that the former equals the latter + the factor (unconditional) means. Given this formulation, inference on factor premiums conditioning on the factors can be performed via the *deltas*, which avoids the enduring error in variable problem that plagues cross-sectional statistics yet preserves the risk premium interpretation underlying these parameters. Our approach is thus to produce confidence sets for the *deltas*, which invert, again, popular tests conditioning on the factors.

To show this, assume for the moment that the above defined deltas are known. Then equilibrium restrictions imply a zero intercept in the MR of excess returns on factors, each in deviation from the corresponding delta. One of the most popular (and for that matter, time series) test for zero intercepts, is the GRS statistic which we duly apply. Where we differ from its classical application is that we apply it for all possible values of the *deltas* in question, keeping non-rejected values. The latter, collected together produce a valid confidence region of dimension equal to the number of factors. Projecting this region onto each "axis" (whose number is equal to the number of factors) produces the reported confidence intervals, which controls the significance level while simultaneously conditioning on the factors in the traditional sense. Then, we proceed to interpreting the derived intervals. Observe that Lewellen et al. (2010) emphasize the usefulness of replacing "tests" with confidence sets building on these tests. One advantage is that confidence intervals provide much more information than tests on the underlying pricing relations. In this paper, we interpret the intervals by checking whether the factor means are covered or not, which informs us on factor pricing.

Let $\theta\,(k \times 1)$ be a vector containing the q structural parameters (δ_j) associated with each j risk factor. Eq. (1) can be written in a more general form where returns on dependent portfolios, r_{it}, are stacked in a matrix $Y\,(T \times n)$, the risk factors, $\tilde{r}_{jt}$, are stacked in a matrix of independent variables X with a column of ones for the constant term (the dimension of X being $T \times n$), and $U\,(T \times n)$ is the matrix of disturbances containing the u_{it}. The disturbances are *i.i.d.* normal with invertible contemporaneous correlation matrix. B is the $k \times n$ matrix of unconstrained sensitivities (b) estimated over each risk factor and the unconstrained intercept (a):

$$B = \begin{bmatrix} a' \\ b \end{bmatrix}, \quad a = (a_1, \ldots, a_n)', \quad b \text{ is } q \times n \tag{14}$$

Then, the model in Eq. (1) can be rewritten in the following general form:

$$Y = XB + U, \tag{15}$$

Let $\hat{B} = (X'X)^{-1}X'Y,\ \hat{S} = \hat{U}'\hat{U},\ \hat{U} = Y - X\hat{B}$. We are interested in the general hypothesis:

$$\mathcal{H}(\theta) : (1, \theta')B = 0, \quad \text{for } \theta \text{ unknown}. \tag{16}$$

When θ is known, the following F-distributed statistic is available:

[7] For a complete derivation of the test statistic when the risk free rate is not observable and a proof of its finite-sample properties, see Beaulieu et al. (2013).

$$\Lambda(\theta)\frac{T-k-n+1}{n} \sim F(n, T-k-n+1), \quad \Lambda(\theta) = \frac{(1,\theta')\hat{B}\hat{S}^{-1}\hat{B}'(1,\theta')'}{(1,\theta')(X'X)^{-1}(1,\theta')'}. \tag{17}$$

Note $\Lambda(\theta)$ is Hotelling's T2 criterion (Hotelling, 1947) which is the multivariate counterpart of the Student-*t* statistic. We apply this testing procedure under the assumption that we can condition on *X*. Normality can be relaxed via classical regression assumptions that validate usual *F*-tests. Alternatively, as in Beaulieu et al. (2010), we also assume that factors pre-whiten errors so *i.i.d.* normal disturbances provide an acceptable working assumption over stable sub-samples. All tests are *invariant to portfolio repacking* in the following sense: the same test decision will result whether the return vector *Y* is used or whether it is transformed into *YA* where A is an $n \times n$ invertible matrix such that $A\iota_n = \iota_n$.[8]

To test for the joint significance of the risk factor and the unconstrained intercept, the test statistic presented in Eq. (17) is used with a conformable specification. For example, the joint significance statistic on the intercept (the GRS statistic) is a special case in which $\theta = 0$. Similarly, a test of joint significance of the first factor in a model would imply replacing θ with a $(k \times 1)$ row vector such as θ = (1,0,0,0,...,0).

In order to test Eqs. (5), (9), and (11), we *invert* the statistic presented in Eq. (17). To invert a test at level α means assembling the set of parameter values not rejected via this test at the α level. The assembled set provides a $1-\alpha$ confidence region for the parameters in question. This region retains the test characteristics of identification-robustness, validity in finite samples, and rotation invariance, as these properties hold for all tested parameter values.

To obtain a confidence set for θ at level $1-\alpha$, we collect all values of θ that are not rejected at the α level by comparing the statistic in (17) to critical values, denoted F_α, for the *F*-distribution. This requires solving, over θ, the inequality

$$\frac{(1,\theta')\hat{B}\hat{S}^{-1}\hat{B}'(1,\theta')'}{(1,\theta')(X'X)^{-1}(1,\theta')'}\frac{\tau_n}{n} \leqslant F_\alpha, \quad \text{with } \tau_n = T-k-n+1. \tag{18}$$

Moving from the joint region defined by (18) to individual confidence sets for each component of θ is achieved by computing, in turn, the smallest and largest values for each parameter included in this region. In the Appendix, we show that all the above can be computed analytically with closed-form formulas.

This solution generates four possible types of confidence sets for the estimated parameters, each having different interpretations: (1) a bounded interval, suggesting that the underlying model is not rejected and does not suffer from identification problems; (2) unbounded sets, implying that the model suffers from identification problems but that the unbounded sets still contain useful information on the parameters (Dufour, 1997); (3) the real line, implying that identification problems are so severe that the data are completely uninformative for the model parameters of interest; (4) an empty set, implying that no value of the parameters in θ exist for which the restriction of the underlying model holds. In this last case, the underlying model is rejected by the data.

3. Data and descriptive statistics

Data are monthly returns for Canada, Germany, Japan and the United Kingdom (UK) which include dividends and capital gains in US dollars. Series are extracted from Datastream, Worldscope and Kenneth French's website.

3.1. Dependent variables

The dependent variables are domestic industry portfolio returns in excess of the lagged one month US T-bill rate (or the one month lagged inflation rate when Black CAPM is used) (r_{it}) for each country. We compute value-weighted returns on portfolios based on the industry classification in Datastream: industrials, consumer goods, financials, oil & gas, technology, consumer services, healthcare, basic materials, utilities and telecommunications. Table 1 presents the descriptive statistics for the industry portfolios across countries.[9,10]

3.2. Factors

Our domestic and global risk factors include market risk and firm-specific factors, as defined in Fama and French (1993 and 2012) and momentum (Carhart, 1997). We also add a foreign exchange risk factor based on Zhang (2006). The global (based on 23 countries) and Japanese risk factors are extracted from Kenneth French's website. We build the Canadian, British and German domestic risk factors following Fama and French (2012). We focus on countries rather than regions to study country financial integration and evaluate the performance of asset pricing models on domestic assets.

The domestic market risk factor (MKT) is the return on a country value-weighted market portfolio minus the US one month lagged T-bill rate. In order to form size (SMB), book-to-market (HML) and momentum (WML) factors, Fama and French (2012) argue that the effect of small stocks has to be mitigated internationally. Therefore, we sort stocks in a country

[8] See Beaulieu et al. (2007, 2010, 2013 and 2015), Kandel and Stambaugh (1989) for earlier perspectives and Roll (2013) for recent financial implication of the concept.

[9] Note that we exclude the German oil industry in the first sub-period, for lack of observations.

[10] We also produced results for size portfolios which were in general not as well identified as the industry ones. Results are available upon request.

Table 1
Descriptive statistic for industry portfolios in excess returns.

	CAN		GER		UK		JP	
Sub-periods	S1	S2	S1	S2	S1	S2	S1	S2
Industrials	.56	.83	−.11	.83	.29	.60	−.37	.32
	(4.70)	(7.26)	(4.89)	(8.83)	(4.96)	(5.96)	(7.64)	(5.96)
	145	199	164	197	444	402	1002	1193
Consumers	.53	.77	.07	.74	.43	1.07	−.02	.18
	(3.71)	(6.00)	(5.47)	(8.03)	(4.38)	(4.43)	(6.06)	(4.66)
	73	75	128	122	234	136	549	618
Financials	1.0	1.06	.44	.26	.91	−.05	−.28	−.16
	(5.37)	(6.42)	(5.03)	(8.37)	(5.07)	(7.54)	(8.35)	(7.22)
	112	196	138	208	387	506	241	325
Oil	.43	1.42	.09	.64	.64	.50	−.24	.47
	(5.43)	(7.79)	(4.84)	(13.14)	(5.87)	(6.80)	(9.25)	(7.53)
	264	419	1	25	34	89	20	20
Technology	1.22	−.31	1.24	.53	1.04	.20	.39	−.23
	(11.62)	(12.99)	(9.85)	(10.74)	(7.58)	(8.92)	(8.72)	(6.28)
	83	150	28	162	104	207	154	382
Service	.73	.46	−.39	.26	.46	.28	−.39	.13
	(4.37)	(5.02)	(5.21)	(8.12)	(4.53)	(5.66)	(6.70)	(4.79)
	87	123	44	112	331	331	354	634
Health	1.32	.08	.79	.91	1.06	.21	.17	.24
	(8.89)	(7.89)	(5.45)	(4.90)	(5.48)	(4.64)	(6.50)	(4.27)
	43	109	17	54	57	109	84	133
Basic	.19	1.46	.16	1.15	.61	1.44	−.37	.42
	(6.22)	(8.89)	(5.19)	(7.73)	(5.99)	(9.70)	(8.18)	(6.21)
	659	1052	54	46	77	136	301	306
Utilities	.58	1.08	.41	.69	1.10	.84	−.12	.16
	(3.69)	(5.25)	(4.40)	(6.67)	(5.69)	(4.71)	(6.23)	(5.11)
	17	31	28	22	30	25	27	30
Telecom	1.13	.88	.79	.20	.94	.42	.34	−.04
	(6.52)	(5.95)	(8.48)	(7.73)	(7.00)	(6.74)	(10.36)	(6.54)
	16	11	5	10	11	25	7	13

Note: This table presents the descriptive statistics of ten domestic (including Canada, Germany, the United Kingdom and Japan) annually-balanced value-weighed industry portfolios. The table reports the monthly mean monthly returns in %, standard errors in parentheses and the average number of stocks in each portfolio per sub-period. S1 (S2) represents the sub-period from 1990–11 to 2001–05 (2001–06 to 2011–12).

into two market capitalization and three book-to-market equity (B/M) groups every year at the end of June. Big stocks are those in the top 90% of the June market capitalization for the country, and small stocks are those in the bottom 10%. The B/M breakpoints are the 30th and 70th percentiles of B/M for the big stocks of a given country. We use the inverse of the price-to-book ratio in Worldscope to obtain B/M for each stock and construct our factors. To construct WML, the size-momentum portfolios are calculated monthly. For portfolios formed at the end of month *t–1*, the lagged momentum return is a stock cumulative return for month *t–12* to month *t–2*. The momentum breakpoints for a region are the 30th and 70th percentiles of the lagged momentum returns of a country big stocks. SMB, HML and WML are obtained using the usual "long minus short" method described on French's website.[11] The forex is defined as the one-month Euro-deposit rate for each of the foreign currencies (CAD, EUR-DEM, GBP and YEN) compounded by the exchange rate variation relative to the US dollar minus the Euro-dollar-deposit rate as described in Zhang (2006).

The data cover the period from November 1990 (the date at which global WML becomes available on French's website) to December 2011. We address possible time-varying sensitivities with a well-established approach (see Black, 1993; Fama and French, 2004 for a survey, Beaulieu et al., 2007 and the references therein). It consists in breaking the sample into sub-periods to provide evidence of a possible time-varying trend in financial market integration.

Table 2 presents descriptive statistics for the domestic and global risk factors for two equal sub-periods. As noted by Fama and French (2012), the standard errors associated with the risk factor averages are large and the significance and magnitude of the risk factors are heterogeneous across countries. The market and forex factors are relatively uniform across countries while firm-specific factors and momentum vary more over time and across countries.

[11] We apply all the filters on the data reported on Kenneth French's website as well as Karolyi and Wu's (2012) filter on Datastream series.

Table 2
Descriptive statistics for domestic and global risk factors.

	Domestic								Global		
	CAN		GER		UK		JP				
Sub-period	S1	S2	S1	S2	S1	S2	S1	S2		S1	S2
r_{MKTDt}	.61	.94	.20	.50	.56	.35	−.30	.03	r_{MKTGt}	.47	.29
	(4.99)	(6.29)	(4.48)	(6.88)	(4.08)	(5.30)	(6.87)	(4.89)		(3.85)	(4.94)
	1.38	1.68	.51	.825	1.54	.75	−.50	6.37		1.38	.69
r_{SMBDt}	−.16	.13	−.86	.13	−.15	.02	−.38	.24	r_{SMBGt}	−.07	.26
	(2.83)	(3.31)	(3.30)	(3.41)	(3.26)	(2.87)	(3.99)	(2.72)		(2.47)	(1.77)
	−.62	.42	−2.93	.43	−.51	.06	−1.07	1.01		−.34	1.67
r_{HMLDt}	.38	.85	.84	.92	.15	.31	.27	.69	r_{HMLGt}	.42	.36
	(5.45)	(3.26)	(3.98)	(3.00)	(2.55)	(2.38)	(3.26)	(2.48)		(2.94)	(1.78)
	.83	2.90	2.37	3.46	.66	1.47	.93	3.13		1.59	2.31
r_{WMLDt}	1.73	.80	.50	.86	1.01	.81	.07	.07	r_{WMLGt}	.75	.49
	(5.72)	(4.63)	(4.06)	(6.57)	(4.63)	(4.45)	(5.40)	(3.84)		(4.12)	(4.17)
	3.4	1.94	1.38	1.47	2.47	2.06	.15	.20		2.04	1.32
r_{forext}	−.18	.39	−.28	.41	−.06	.21	−.12	.22	μ_{rf}	.39	.16
	(1.37)	(2.85)	(3.10)	(3.17)	(2.80)	(2.64)	(.036)	(2.73)		(.09)	(.14)
	−1.49	1.56	−1.00	1.45	−.23	.90	−.38	.89			

This table presents descriptive statistics for the domestic and global risk factors. Means and their associated standard errors in parentheses are presented with their t statistic underneath. r_{MKTDt} represents the domestic value-weighted market index return in excess of the global risk free rate and r_{MKTGt} is the monthly global market excess return. r_{SMBDt}, r_{HMLDt} and r_{WMLDt} are respectively the domestic, book-to-market and momentum risk factors while r_{SMBGt}, r_{HMLGt} and r_{WMLGt} are their global counterpart. The global and Japanese risk factors are extracted from Kenneth French's website while the Canadian, German and British domestic risk factors are constructed following Fama and French (2012). The data are extracted from Datastream and Worldscope. r_{forext} is the exchange rate risk factor obtained as the monthly excess return in % (over the US Euro-dollar) of foreign currency holdings as in Zhang (2006). For Germany, we use the DM exchange rate until December 1998 and the Euro exchange rate afterward. Exchange rates are extracted from Bloomberg. Finally, the mean of the global risk free rate, μ_{rf}, is also reported. It was extracted on Kenneth French's website. S1 (S2) is from 1990–11 to 2001–05 (2001–06 to 2011–12). All risk factors are expressed in US dollar returns.

4. Results

Tables 3 to 6 present the results on the empirical factor pricing models for the domestic, global and international specifications using industry portfolios as dependent variables. The evidence on domestic, global and international factor models and Black CAPM highlights the heterogeneity in factor and model selection at the international level.

4.1. Empirical factor pricing models

We now present results for several empirical factor models under three different specifications: domestic (Eq. (4)), global (Eq. (6)) and international (Eq. (8)). In all specifications, each risk factor is constrained to zero in turn, in a step by step analysis, to investigate the marginal contribution of each factor for model fit as well as for the potential source of identification problems.

4.1.1. Domestic factor pricing models

Table 3 presents results for the domestic empirical factor pricing models. Results show that domestic models are well-identified in the presence of less than four factors but that adding momentum creates identification problems in all countries except the UK. All the sensitivities are jointly significant. In terms of the intercept, the significance of the GRS statistic depends on the country and sub-period under study rather than on the choice of specific factors. In fact, the GRS statistic shows that the intercepts are significant in both sub-periods in the UK and Germany (except one case in the first sub-period) independently of the specification of the domestic risk. In Canada, the intercepts are insignificant for all specification for the first sub-period, while the results from the second sub-period reveal that the intercepts are insignificant for the three factor model. In Japan, the intercepts are never significant in any model specification or sub-period.

4.1.2. Global factor pricing models

Table 4 presents results for the global risk factor models. First, the GRS statistic is not significant in Canadian and Japanese models. In the UK, the intercepts are significant, except for the models with fewer factors in the first sub-period. For Germany, the intercept in several models are insignificant while the three factor model (with and without forex) intercepts are significant as well as for the market model augmented with forex. The GRS statistics for the global models present an equal or better fit than their domestic counterpart. However, identification problems are more important with the global specification (shown by confidence intervals for δs that cover the real line, indicating that any value of δ is consistent with the model) than for the domestic one. More specifically, the global momentum risk factor is not significant in Germany and

Table 3
Results for the domestic models.

	δ^{MKTD}	δ^{SMBD}	δ^{HMLD}	δ^{WMLD}	H_0: $a_1 = 0$	H_0: $\beta_i^{SMBD} = 0$	H_0: $\beta_i^{HMLD} = 0$	H_0: $\beta_i^{WMLD} = 0$
Panel A: Canada								
90–01	$-\infty, -1.74 \cup -.82, \infty$	$-\infty, -4.86 \cup -2.91, \infty$	$\mathbb{R}$	$-\infty, -20.01 \cup 10.04, \infty^*$	.66	.00	.00	.00
	(−.77, .63)	(.77, 1.18)*	(−.95, .63)	(−1.88, 3.22)				
01–11	[−.08, .71]*	[−3.81, 2.4]	[−2.65, 1.12]	[−2.07, 3.99]	.02	.00	.00	.00
	(.11, .38)*	(−1.14, .69)	(−1.25, .03)*	(.73, 1.46)				
90–01	[−.47, .67]	[−2.57, 2.01]	[−1.89, 1.18]		.70	.00	.00	
	(−.12, .36)*	(−.97, .69)	(−1.08, .24)*					
01–11	[−.06, .38]*	[−2.24, 2.42]	[−1.91, 1.08]		.11	.00	.00	
	(.06, .27)*	(−.84, .87)	(−1.11, .11)*					
90–01	[−.43, .67]	[−2.49, 2.00]			.71	.00		
	(−.10, .37)*	(−.95, .70)						
01–11	[−2.05, 1.37]	[2.05, 1.37]			.04	.00		
	(.09, .29)*	(−.99, .52)						
90–01	[−.24, .58]*				.69			
	(−.37, .04)*							
01–11	[.01, .35]*				.04			
	(.08, .28)*							
Panel B: United Kingdom								
90–01	[.00, .88]	[−2.80, 1.75]	[−2.09, 4.06]	[−3.51, 9.66]	.00	.00	.00	.00
	(.15, .40)*	(−1.28, .19)	(−.70, .55)	(−.91, 1.75)				
01–11	[.09, .50]	[−.65, 1.17]	[−1.16, .71]	[−.63, 3.23]	.00	.00	.00	.00
	(.17, .36)	(−.24, .61)	(−.73, .11)*	(.05, 1.76)				
90–01	[.03, .42]*	[−2.64, .90]	[−1.75, .58]		.01	.00	.00	
	(.12, .31)*	(−1.29, .14)	(.93, .06)*					
01–11	[.13, .24]*	[.26, .31]*	[−.84, .27]*		.00	.00	.00	
	(.06, .28)*	(−.84, .87)	(−1.11, .11)*					
90–01	[.03, .36]*	[−1.75, .87]			.01	.00		
	(.11, .28)*	(−1.00, .26)						
01–11	[.15, .21]*	[−.09, .23]			.00	.00		
	(.10, .26)*	(−.30, .44)						
90–01	[.04, .35]*				.01			
	(.11, .28)*							
01–11	[.16, .21]*				.00			
	(.11, .26)*							
Panel C: Japan								
90–01	$\mathbb{R}$	$\mathbb{R}$	$-\infty, -4.29 \cup -2.01, \infty$	$-\infty, -6.24 \cup -1.90, \infty$	.21	.00	.00	.00
	(.02, .51)*	(.83, 1.19)	(−.61, 1.77)	(.48, 4.07)*				
01–11	$\mathbb{R}$	$\mathbb{R}$	$\mathbb{R}$	$\mathbb{R}$	.46	.00	.00	.00
	(.07, .62)*	(−1.91, .14)	(−.82, 1.05)	(.03, 5.03)				
90–01	[−.17, .56]*	[−2.15, 1.15]	[−1.87, 1.70]		.21	.00	.00	
	(.01, .40)*	(−1.10, .42)	(−1.01, .55)					
01–11	[−.07, .34]	[−1.39, 1.11]	[−1.63, 2.33]		.37	.00	.00	
	(.02, .22)	(−.73, .38)	(−.78, .66)*					

(*continued on next page*)

Table 3 (*continued*)

	δ^{MKTD}	δ^{SMBD}	δ^{HMLD}	δ^{WMLD}	H_0: $a_1 = 0$	H_0: $\beta_i^{SMBD} = 0$	H_0: $\beta_i^{HMLD} = 0$	H_0: $\beta_i^{WMLD} = 0$
90–01	[−.08, .53]*	[−1.55, .88]			.17	.00		
	(.05, .40)*	(−.93, .39)						
01–11	[−.06, .28]	[−1.26, .88]			.27	.00		
	(.02, .21)	(−.73, .33)						
90–01	[−.06, .46]*				.13			
	(.03, .37)*							
01–11	[−.06, .28]				.26			
	(.02, .21)							
Panel D: Germany								
90–01	$\mathbb{R}$	$\mathbb{R}$	$\mathbb{R}$	$\mathbb{R}$	.02	.00	.00	.00
	(.14, .77)	(−.67, 1.66)*	(−1.45, .82)*	(.31, 4.47)				
01–11	[−.04, .60]	[−1.33, 2.64]	[−3.78, .79]*	[−1.73, 3.33]	.00	.00	.00	.00
	(.10, .41)*	(−.57, 1.04)	(−2.08, −.25)*	(−.82, 1.52)				
90–01	[−.08, .39]	[−3.04, .75]	[−2.82, −.01]*		.02	.00	.00	
	(.00, .30)	(−1.52, .33)	(−2.08, .43)*					
01–11	[−.02, .36]*	[−1.16, 1.23]	[−2.61, .36]*		.00	.00	.00	
	(.04, .30)*	(−.75, .69)	(−1.91, −.21)*					
90–01	[−.04, .31]	[−.07, .68]*			.08	.00		
	(−.01, .28)	(−1.11, .49)						
01–11	[.04, .34]*	[−1.14, 1.05]			.02	.00		
	(.07, .30)*	(−.78, .63)						
90–01	[.06, .20]*				.02			
	(.00, .27)							
01–11	[.07, .30]*				.01			
	(.07, .30)*							

This table presents confidence sets from the estimates of Eq. (1) imposing Eq. (4) for several domestic factors in the first four columns. They are obtained from the inverted Hotelling statistic (Eq. (18)) while those in parentheses are obtained from the Wald statistic (A.11). The last four columns report the *p*-values for the hypotheses presented in Eq. (2) and Eq. (3). The Hotelling *p*-values for the significance of the market risk factor are not presented since they are all significant. Significance is denoted by a * and reported at the 5% level.

Table 4
Results for the global models.

	δ^{MKTG}	δ^{SMBG}	δ^{HMLG}	δ^{WMLG}	δ^{FOREX}	H_0: $a_1 = 0$	H_0: $\beta_i^{SMBG} = 0$	H_0: $\beta_i^{HMLG} = 0$	H_0: $\beta_i^{WMLG} = 0$	H_0: $\beta_i^{forex} = 0$
Panel A: Canada										
90–01	ℝ	ℝ	ℝ	ℝ	ℝ	.53	.00	.00	.06	.00
	(−.72, 1.59)	(−.86, .73)	(−1.26, .21)*	(−1.01, 2.79)	(−.32, .81)					
01–11	ℝ	ℝ	ℝ	ℝ	ℝ	.59	.04	.00	.00	.00
	(−.52, .74)	(−.61, 1.20)	(−.53, .54)	(−1.44, 1.43)	(−.25, .73)					
90–01	ℝ	ℝ	ℝ	ℝ		.79	.00	.00	.06	
	(−.28, 1.11)	(−.80, .67)	(−1.18, .11)*	(−.98, 2.74)						
01–11	ℝ	ℝ	ℝ	ℝ		.61	.03	.00	.00	
	(−.10, .71)	(−.50, 1.31)	(−.35, .61)	(−.93, 1.59)						
99–01	ℝ	ℝ	ℝ		ℝ	.31	.00	.00		.00
	(−.92, 1.08)	(−.78, .67)	(−1.37, .07)*		(−.04, .93)					
01–11	ℝ	ℝ	ℝ		ℝ	.54	.02	.00		.00
	(−.45, .53)	(−.55, 1.22)	(−.53, .49)		(−.15, .72)					
90–01	[−1.02, 2.11]	[−2.39, 1.62]	[−2.32, .84]			.56	.00	.00		
	(−.27, .99)	(−.83, .51)	(−1.20, −.03)*							
01–11	[−3.36, 1.25]	[−1.21, 33.64]	[−.89, 9.50]			.52	.01	.00		
	(−.16, .62)	(−.16, 1.55)	(−.37, .65)							
90–01	ℝ				ℝ	.23				.00
	(−.79, 12.67)				(.00, .82)*					
01–11	[−1.31, .85]				[−.31, 1.49]	.33				.00
	(−.53, .39)				(.07, .81)					
90–01	[−.95, 1.93]	[−1.76, 1.22]				.82	.00			
	(−.27, .98)	(−.75, .45)								
01–11	[−1.01, .95]	[−1.16, 8.16]				.39	.02			
	(−.14, .60)	(−.19, 1.37)								
90–01	[−.85, 1.60]					.82				
	(−.28, .87)									
01–11	[−.24, .84]					.26				
	(−.04, .63)									
Panel B: United Kingdom										
90–01	ℝ	ℝ	ℝ	ℝ	ℝ	.01	.00	.00	.02	.00
	(−.36, 1.57)	(−1.77, .17)	(−1.14, .18)*	(−.94, 4.48)	(−1.00, 1.32)					
01–11	ℝ	ℝ	ℝ	ℝ	ℝ	.02	.00	.00	.00	.00
	(−.70, .53)	(−.27, .81)	(−.99, .09)*	(−.01,1.94)	(−.24, .186)					
90–01	ℝ	ℝ	ℝ	ℝ		.03	.00	.00	.02	
	(−.19, 1.21)	(−1.67, .15)	(−1.12, .16)*	(−.77, 3.99)						
01–11	[−.39, .98]	[−.85, 1.36]	[−1.38, .48]	[−.44, 3.76]		.02	.00	.00	.00	
	(−.08, .61)	(−.37, .58)	(−.87, −.03)*	(.25, 2.11)						
99–01	[−2.16, 2.66]	[−2.03, 1.12]	[−1.81, 1.12]		[−2.44, 3.92]	.00	.00	.00		.00
	(−.38, 1.08)	(−.94, .30)	(−.124, .42)*		(−.31, 1.40)					
01–11	[−6.81, .42]	[−.56, 6.07]	[−3.19, .43]		[−.11, 14.01]	.01	.00	.00		.00
	(−1.19, .08)*	(−.17,1.10)	(−1.17, .15)*		(.51, 2.76)*					

(*continued on next page*)

Table 4 (*continued*)

	δ^{MKTG}	δ^{SMBG}	δ^{HMLG}	δ^{WMLG}	δ^{FOREX}	H_0: $a_1 = 0$	H_0: $\beta_i^{SMBG} = 0$	H_0: $\beta_i^{HMLG} = 0$	H_0: $\beta_i^{WMLG} = 0$	H_0: $\beta_i^{forex} = 0$
90–01	[−.67, 1.80] (.06, 1.08)	[−1.99, 1.09] (−.95, .29)	[−1.72, −.06]* (−1.27, −.48)*			.00	.00	.00		
01–11	[−.06, .38] (−.14, .47)	[−.32, −.38] (−.40, .44)	[−.79, −.18]* (−.86, −.11)*			.01	.00	.00		
90–01	[−1.53, 2.56] (−.24, 1.17)				[−2.24, 3.25] (−.35, 1.33)	.20				.00
01–11	[−1.54, .37]* (−.80, .18)*				[−.04, 3.56] (.35, 2.05)*	.03				.00
90–01	[−.52, 1.79] (.00, 1.13)	[−1.70, 1.08] (−.85, .35)				.38	.00			
01–11	[−.01, .37] (−.12, .48)	[−.26, .35] (−.38, .45)				.05	.00			
90–01	[−.42, 1.78] (.05, 1.15)					.39				
01–11	[.13, .29]* (−.08, .51)					.04				
Panel C: Japan										
90–01	$\mathbb{R}$ (−1.55, .37)*	$\mathbb{R}$ (−.22, 3.91)	$\mathbb{R}$ (−1.08, 1.70)	$\mathbb{R}$ (−.44, 4.48)	$\mathbb{R}$ (−1.09, 1.03)	.53	.03	.00	.00	.00
01–11	$\mathbb{R}$ (−1.27, 3.55)	$\mathbb{R}$ (−2.62, .50)	$\mathbb{R}$ (−.76, 1.52)	$\mathbb{R}$ (−1.23, 5.77)	$\mathbb{R}$ (−1.21, .70)	.73	.00	.00	.02	.00
90–01	$\mathbb{R}$ (−1.39, .33)*	$\mathbb{R}$ (−.19, 3.77)	$\mathbb{R}$ (−1.03, 1.71)	$\mathbb{R}$ (−.39, 4.25)		.50	.03	.00	.00	
01–11	$\mathbb{R}$ (−1.31, 3.62)	$\mathbb{R}$ (−2.65, .52)	$\mathbb{R}$ (−.74, 1.57)	$\mathbb{R}$ (−1.21, 5.94)		.73	.00	.00	.03	
99–01	$\mathbb{R}$ (−1.04, .32)*	$\mathbb{R}$ (−.13, 2.46)	$\mathbb{R}$ (−1.16, .62)		$\mathbb{R}$ (−.87, .68)	.22	.13	.00		.00
01–11	[−7.12, 4.97] (−1.44, .48)	[−3.26, −4.20] (−.54, .61)	[−4.20, 3.97] (−.74, .57)		[−1.67, 1.39] (−.62, .58)	.69	.00	.00		.00
90–01	$\mathbb{R}$ (−1.05, .20)*	$\mathbb{R}$ (−.06, 2.53)*	$\mathbb{R}$ (−1.19, .52)			.18	.35	.00		
01–11	[−6.37, 4.43] (−1.46, .45)	[−2.87, 3.67] (−.50, .62)	[−3.92, 3.76] (−.72, .58)			.67	.00	.00		
90–01	[−1.30, .83] (−.87, .33)*				[−1.22, .99] (−.69, .54)	.35				.00
01–11	[−4.04, 1.35] (−1.59, .21)*				[−1.15., 2.38] (−.39, .70)	.53				.00
90–01	[−1.47, .68] (−.93, .21)*	[−1.89, 6.86] (−.25, 1.77)				.29	.02			
01–11	[−.06, .28]* (.02, .21)*	[−1.26, .88] (−.73, .33)				.27	.00			
90–01	[−1.23, .55] (−.90, .20)*					.30				
01–11	[−1.47, .96] (−.91, .39)					.58				

Panel D: Germany

90–01	ℝ	ℝ	ℝ	ℝ	ℝ	.08	.02	.00	.12	.00
	(−5.04, 1.82)	(−2.24, 9.76)	(−1.74, 1.29)	(−3.71, 18.51)	(−1.50, 6.25)					
01–11	ℝ	ℝ	ℝ	ℝ	ℝ	.04	.00	.00	.03	.00
	(−.65, .66)	(−.48, 1.07)	(−2.15, −.33)*	(−1.05, 2.41)	(−.68, 2.39)					
90–01	ℝ	ℝ	ℝ	ℝ		.09	.03	.00	.12	
	(−2.22, 1.64)	(−1.59, 7.83)	(−1.59, 1.12)	(−2.53, 16.31)						
01–11	ℝ	ℝ	ℝ	ℝ		.14	.05	.00	.12	
	(−2.10, 1.53)	(−1.31, 7.34)	(−1.52, 1.03)	(−1.87, 14.32)						
99–01	ℝ	ℝ	ℝ		ℝ	.04	.06	.00		.00
	(−236.42, 279.00)	(−691.61, 585.54)	(−88.62, 74.22)		(−398.69, 338.64)					
01–11	ℝ	ℝ	−∞, .09 ∪ 4.04, ∞*		−∞, −7.08 ∪ −1.40, ∞	.02	.00	.00		.00
	(−.78, .44)	(−.45, 1.23)	(−2.36, −.55)		(−.10, 2.69)*					
90–01	ℝ	ℝ	ℝ			.04	.07	.00		
	(−.86, 1.36)	(−4.73, −.10)*	(−1.69, −.07)*							
01–11	[−.41, 1.24]	[−.91, 4.30]	[−4.48, .04]*			.02	.00	.00		
	(−.07, .67)	(−.28, 1.09)	(−1.74, −.46)*							
90–01	[.03, .22]*				[.04, 1.77]*	.02				.00
	(−.01, .26)*				(−.05, 1.78)*					
01–11	[−.11, .33]				[−.91, 3.48]	.03				.00
	(.01, .27)*				(−.19, 1.65)					
90–01	[−1.12, 1.06]	[−.59, 1.85]				.06	.00			
	(−.95, .90)	(−.34, 1.57)								
01–11	[−.68, .62]	[−.75, 2.63]				.12	.00			
	(−.34, .47)	(−.27, 1.52)								
90–01	[−.38, .75]					.09				
	(−.58, .91)									
01–11	[−.12, .66]					.12				
	(−.04, .58)									

This table presents the estimates of Eq. (1) imposing Eq. (6) for several global factors. The first five columns report the confidence sets obtained for each factor estimates. They are obtained from the inverted Hotelling statistic (Eq. (18)) while those in parentheses are obtained from the Wald statistic (A.11). The last five columns report *p*-values for the hypotheses in Eqs. (2) and (3). The Hotelling *p*-values for the significance of the market risk factor are not presented since they are all significant. Significance is denoted by a * and reported at the 5% level.

Table 5
Results for the international models.

	δ^{MKTG}	$\delta^{MKTD\perp}$	δ^{SMBG}	$\delta^{SMBD\perp}$	δ^{HMLG}	$\delta^{HMLD\perp}$	δ^{WMLG}	$\delta^{WMLD\perp}$	δ^{forex}
Panel A: Confidence sets on the coefficients for Canada									
90–01	ℝ	ℝ	ℝ	ℝ	ℝ	ℝ	ℝ	ℝ	ℝ
	(−24.38, 29.75)	(−18.80, 17.45)	(−18.58, 15.48)	(−94.41, 114.16)	(−.68.27, 54.86)	(−36.13, 45.17)	(−41.01, 35.40)	(−.83.09, 101.24)	(−15.87, 19.90)
01–11	ℝ	ℝ	ℝ	ℝ	ℝ	ℝ	ℝ	ℝ	ℝ
	(−47.76, 45.32)	(−32.74, 35.78)	(−.789.69, 739.15)	(−284.75, 302.21)	(−446.65, 416.98)	(−646.87, 691.47)	(−456.22, 425.37)	(−360.41, 386.77)	(−149.86, 140.24)
90–01	ℝ	ℝ	ℝ	ℝ	ℝ	ℝ	ℝ	ℝ	
	(−45.26, 52.35)	(−27.46, 25.76)	(−40.77, 35.54)	(−200.23, 229.13)	(−130.30, 112.23)	(−71.61, 83.47)	(−75.90, 67.79)	(−178.83, 205.48)	
01–11	ℝ	ℝ	ℝ	ℝ	ℝ	ℝ	ℝ	ℝ	
	(−1.41, 1.94)	(−1.06, 2.24)	(−2.29, 3.21)	(−3.95, 1.04)	(−2.27, 1.48)	(−2.90, 2.39)	(−3.51, 1.70)	(−2.14, 4.99)	
90–01	ℝ	ℝ	ℝ	ℝ	ℝ	ℝ			ℝ
	(−5.69, 13.00)	(−11.85, 5.71)	(−6.76, 2.89)	(−3.77, 7.09)	(−9.95, 3.09)	(−3.93, 11.05)			(−.73, 1.79)
01–11	ℝ	ℝ	ℝ	ℝ	ℝ	ℝ			ℝ
	(−.98, 1.41)	(−.56, 1.90)	(−3.15, 7.19)	(−6.14, 2.15)	(−2.50, 3.43)	(−6.14, 3.24)			(−2.36, 3.32)
90–01	ℝ	ℝ	ℝ	ℝ	ℝ	ℝ			
	(−5.88, 13.24)	(−12.04, 5.88)	(−6.75, 2.94)	(−3.89, 7.30)	(−10.39, 3.32)	(−4.13, 11.40)			
01–11	ℝ	ℝ	ℝ	ℝ	ℝ	ℝ			
	(−3.73, 4.98)	(−3.74, 4.37)	(−23.07, 33.68)	(−22.87, 14.91)	(−13.17, 17.74)	(−32.05, 22.51)			
90–01	[−1.44, .87]	[−.20, 2.44]							[−.95, 1.80]
	(−1.67, 2.21)	(−1.97, 2.05)							(−.01, .82)*
01–11	ℝ	ℝ							ℝ
	(−.63, .34)	(.41, 1.52)*							(−.18, .75)
90–01	ℝ	ℝ	ℝ	ℝ					
	(−4.02, 1.23)	(−.97, 4.36)	(−.62, 1.52)	(−1.79, .58)					
01–11	ℝ	ℝ	ℝ	ℝ					
	(−.59, .62)	(−.15, 1.51)	(−.65, 2.05)	(−2.34, .28)					
90–01	ℝ	ℝ							
	(−2.12, 1.65)	(−1.40, 2.51)							
01–11	[−.17, 2.39]	[−.17, 2.39]							
	(−.61, .34)	(.41, 1.51)*							
	H_0: $a_i = 0$	H_0: $\beta_i^{MKTD\perp} = 0$	H_0: $\beta_i^{SMBG} = 0$	H_0:$\beta_i^{SMBD\perp} = 0$	H_0: $\beta_i^{HMLG} = 0$	H_0: $\beta_i^{HMLD\perp} = 0$	H_0: $\beta_i^{WMLG} = 0$	H_0: $\beta_i^{WMLD\perp} = 0$	H_0: $\beta_i^{forex} = 0$
Panel B: Hotelling statistics for joint significance test on each factor sensitivity in Canada									
90–01	.19	.00	.00	.00	.00	.00	.00	.00	.04
01–11	.00	.00	.04	.00	.00	.00	.00	.00	.00
90–01	.19	.00	.00	.00	.00	.00	.00	.00	
01–11	.00	.00	.03	.00	.00	.00	.00	.00	
90–01	.23	.00	.00	.01	.00	.00			.03
01–11	.00	.00	.03	.00	.00	.00			.00
90–01	.23	.00	.00	.00	.00	.00			

01–11	.00	.00	.02	.00	.00	.00			
90–01	.16	.00							.01
01–11	.00	.00							.00
90–01	.18	.00	.00	.00					
01–11	.00	.00	.02	.00					
90–01	.24	.00							
01–11	.00	.00							

This table presents the estimates of Eq. (1) imposing Eq. (8) for several specification of international systematic risk. Panel A reports the confidence sets obtained for each risk factor estimate. On the first line, they are obtained from the inverted Hotelling statistic (Eq. (18)) while those presented in parentheses are obtained from the Wald statistic (A.11). Panel B reports the Hotelling *p*-values for the joint test presented in Eqs. (2) and (3). The Hotelling *p*-values for the market risk factor are omitted since they are all significant. Significance is denoted by a * and reported at the 5% level.

Table 6
Results for Black models.

	International			Domestic		Global	
	γ	$\delta^{MKTD\perp}$	H_0: $a_i = 0$	γ	H_0: $a_i = 0$	γ	H_0: $a_i = 0$
Panel A: Canada							
90–01	[−.99, 1.69]	[−.16, 0.76]	.16	[−.69, 1.77]	.58	[−.88, 1.67]	.74
	(−.22, .99)	(−.51, .07)*		(−.05, 1.16)		(−.18, 1.02)	
01–11	[−2.24, 2.79]	[.50, 1.20]*	.00	[∅]	.04	[−1.75, 1.29]	.26
	(−.84, 1.28)	(−1.00, −.68)*		(−1.54, .71)		(−1.19, .82)	
90–95	[−2.88, 1.77]	[−.67, 1.67]	.00	[−2.78, 1.74]	.91	[−1.80, 1.55]	.89
	(−.88, .65)	(−.67, .10)		(−.98, 62)		(−.63, .69)	
96–01	[−1.04, 2.72]	[−.14, 1.50]	.99	[−.80, 2.55]	.32	[− .98, 2.20]	.36
	(−.06, 1.58)	(.26, 1.01)*		(.03, 1.72)		(−.18, 1.40)	
01–06	[−1.58, 4.09]	[.76, 1.34]*	.00	[∅]	.00	[.64, 2.74]*	.03
	(−.03, 2.16)	(−1.19, −.93)*		(1.76, 4.72)*		(.56, 2.67)*	
06–11	[−5.21, 4.53]	[−.38, 1.42]	.00	[−4.04, 2.04]	.42	[−4.99, 3.00]	.74
	(−1.85, 1.26)	(−.82, −22)*		(−.27, .68)		(−2.14, 0.81)	
Panel B: United Kingdom							
90–01	[−5.94, 3.18]	[−.26, 1.65]	.00	∅	.01	[−2.02, 3.00]	.38
	(−1.54, 1.21)	(−.73, −.15)*		(−4.10, −.11)*		(−.62, 1.68)	
01–11	[−.12, 1.27]	[.17, .29]*	.00	∅	.00	[.46, .79]*	.04
	(−.24, 1.37)	(−.30, −.15)*		(.61, 2.38)*		(−.18, 1.42)	
90–95	ℝ	ℝ	.00	ℝ	.24	ℝ	.39
	(−.31, 12.90)	(−.22, .02)		(.99, 7.69)*		(−.33, 13.06)	
96–01	[−3.11, 3.35]	[−.55, 1.46]	.00	[−2.35, 1.47]	.22	[−1.47, 3.04]	.70
	(−.58, 1.45)	(.68, .03)*		(−1.23, .81)		(−.14, 1.64)	
01–06	[−1.18, 2.06]	[.22, .57]*	.00	∅	.02	[−.48, 1.82]	.12
	(−.34, 1.28)	(−.49, −.30)*		(.00, 2.20)		(−.13, 1.43)	
06–11	[−1.58, 2.61]	[−.07, .19]	.14	∅	.01	[−1.48, 2.49]	.17
	(−.83, 1.75)	(−.16, .03)		(.43, 2.98)*		(−.83, 1.75)	
Panel C: Japan							
90–01	[−3.04, 2.15]	[−1.72, −.09]*	.00	[−.86, 1.79]	.13	[−1.55, 2.11]	.26
	(−1.20, 1.04)	(−.43, 1.17)		(−.75, 1.63)		(−.67, 1.37)	
01–11	[−.92, 1.98]	[−.84, .32]	.49	[−.58, 1.89]	.26	[−.87, 1.67]	.60
	(−.22, 1.20)	(−.04, .53)		(−.12, 1.35)		(−.26, 1.05)	
90–95	ℝ	ℝ	.00	[−4.17, 4.25]	.86	[−2.00, 3.26]	.91
	(−1.23, 1.30)	(−.10, 1.94)		(−1.54, 1.67)		(−.22, 1.50)	
96–01	[−2.21, 3.79]	[−1.1, .20]	.01	[−.27, 2.40]	.07	[−1.94, 3.45]	.26
	(−.50, 1.96)	(−.38, .91)		(−.32, 2.34)		(−.50, 1.94)	
01–06	[−1.47, 3.63]	[−1.19, .75]	.19	[−1.24, 2.57]	.56	[−1.27, 3.17]	.50
	(−.06, 1.83)	(−.22, 52)		(−.22,1.49)		(−.06, 1.73)	
06–11	[−1.78, 3.67]	[−1.82, .44]	.02	[−1.17, 3.89]	.34	[−1.64, 2.14]	.58
	(−.44, 1.70)	(−.12, 1.01)		(−.15, 2.12)		(−.68, 1.14)	
Panel D: Germany							
90–01	[−3.40, 0.99]	[−.26, .93]	.10	∅	.02	[−1.71, .77]	.09
	(−2.31, .74)	(−.65, 21)		(−.43, 1.99)		(−1.59, .83)	
01–11	[−.68, 1.17]	[.09, .64]*	.00	∅	.02	[−.63, .60]	.12
	(−.46, .93)	(.−57, −.15)*		(.03, 1.85)		(−.65, .63)	
90–95	−∞, −.49 ∪ 6.08, ∞*	−∞, −2.62 ∪ .23, ∞*	.00	ℝ	.00	[−40.29, −.62]*	.02
	(−15.31, .63)	(−6.65, .26)		(−133.44, 66.15)		(−6.31, −1.25)*	
96–01	[−5.23, 2.24]	[−.52, 1.05]	.70	[−2.61, 2.04]	.53	[−4.65, 2.11]	.71
	(−1.95, .67)	(−.44, 20)		(−1.31, 0.98)		(−1.84, .65)	
01–06	[−1.04, 4.05]	[−.08, 1.96]	.00	[.38, 3.14]*	.03	[−.99, 1.51]	.26
	(.05, 1.89)	(−1.10, −.35)*		(.60, 2.82)*		(−.45, 1.01)	
06–11	[−.86, 1.72]	[−.04, .65]	.07	[−.48, 2.37]	.13	[−.81, 1.45]	.21
	(−.38, 1.19)	(−.51, −.08)*		(−.04, 1.88)		(−.45, 1.07)	

Notes: This table presents the confidence sets and p-values resulting from the estimation of the international, domestic, and global BCAPM models. In this model, the estimation of Eq. (1) is performed on returns in excess of the one-month US inflation rate. The bracket confidence sets are obtained from the inverted Hotelling statistic Eq. (18) while the confidence intervals in parentheses are obtained from the Wald statistic (A.11). The Hotelling statistic p-values for the tests of joint significance of the intercept (Eq. (2)) are also reported. The Hotelling test p-values for the joint significance of the market risk factor Eq. (3) are omitted, since they are all significant. Test significance at the 5% level is denoted by *.

Canada in the first sub-period while size is not always significant in Germany and Japan. Such mixed evidence on the usefulness of momentum is an international illustration of the result found in Beaulieu et al. (2010) where momentum is redundant when added to the Fama and French model in the US. Japan and Germany provide a good illustration of why the GRS statistic used alone can be misleading, since it is often insignificant. This suggests that models are well-specified when in fact the inverted Hotelling statistic shows that the model is not well identified.

4.1.3. International factor pricing models

Table 5 presents results for international empirical factor models, *i.e.* models in which both global and domestic sources of risk are present. These models permit a test of financial integration of one country domestic risk factors with global financial markets as defined in Eq. (9). For brevity, we only report the Canadian evidence. Our results show that international models with many factors suffer from important identification problems. Excepted for the international market model and in one period the market model supplemented with forex, none of the other specifications are well-identified which implies that integration tests based on these models are fallacious. With a different approach, Zhang's (2006) evidence supports a market ICAPM including an exchange risk factor. Our results show that this model is not always identified. These results underscore the pitfalls of using multiple sources of "similar" risk.

Hou et al. (2011) document the relative importance of country-specific versus global factor models. They find that the international version of the models considered outperform in most countries the purely global models. Griffin (2002) finds that the international three factor model is superior to the global model, but that the domestic model performs even better. Our analysis complements these findings by showing that domestic factor models with few factors are better identified than the other empirical risk factor models. Our identification-robust results suggest that international empirical factor models should not be used without first assessing whether they present identification problems. Finally, note that although the international model provides a framework for tests of financial integration, the tests are not informative with our data due to the lack of identification of such models.

4.2. Black CAPMs

Our results with empirical factor models highlight important identification problems, the difficulty to choose the appropriate factors and the impossibility to provide a reliable test of financial integration. In that context, we turn to a parsimonious model that imposes Black's constraint to the international market model (Mittoo, 1992). Her international BCAPM has a built-in test for integration. The domestic equation (12) and global equation (13) counterpart of the model are estimated as well. Table 6 presents the results for our two sub-periods. We also present evidence for sub-periods of five years. Studying shorter sub-periods enables us to document possible time-trends in financial market integration. To assess model specification, we test the hypothesis presented in Eq. (2). The unconstrained intercepts are jointly significant in more than half the cases for our four countries in the international and domestic models, but the unconstrained intercepts are not significant for almost every sub-period and country for the global models. Overall, the evidence reveals that the models considered are well-identified and the confidence sets are compact, except in the 90–95 sub-periods where identification problems are found in three out of the four countries studied.

We test for financial market integration through the international model via Eq. (11), a test on the purely domestic risk premium $\delta^{MKTD\perp}$ using the inverted Hotelling statistic. For Canada, the UK and Germany, financial market integration is not rejected in the first ten year sub-period while it is rejected in the second one. We find the opposite for Japan. Since most countries experienced stock market growth during this period and not Japan, this is not a surprising result. Over five year sub-periods, we find variations at the country level in financial integration tests. Our results show no evidence of a time-trend toward financial integration. Rather, the significance of the purely domestic residuals seems to be time-varying. An important strand of the literature suggests time-varying financial integration (e.g. Bekeart and Harvey, 1995; Carrieri et al., 2007 and Berger and Pozzi, 2013). Our empirical findings confirm that a trend towards integration is refuted while a time varying integration process is suggested. Finally, although the model is well-identified, we find several changes of decision when looking at Wald confidence sets compared to our proposed Hotelling inverted test ones. Test outcomes for financial integration are reversed in 9 cases out of 24. While the inverted Hotelling statistic provides reliable inference and diagnoses identification problems (Beaulieu et al., 2013), this finding illustrates the relevance of the proposed inverted statistic in an international context. The evidence further calls for caution in interpreting traditional Wald test results which are misleading even when the model appears to be well-identified (see Dufour and Dagenais, 1992 and Beaulieu et al., 2013).

The domestic model is often rejected in Canada, the UK and Germany, as the confidence set associated with γ is empty in several sub-periods. Empty confidence sets are not a symptom of identification problem, but rather that the constraint imposed on the intercept is rejected by the data. Thus, these results suggest that imposing Black's constraint on a domestic model is not appropriate in our sample and since Wald confidence intervals are tight in this model, this problem would not be diagnosed with a testing methodology that does not provide a built-in check for model specification.

5. Discussion and conclusion

In this paper, we estimate several popular international asset pricing models and test for financial integration using an identification-robust estimation approach valid in finite samples. Our methodology has a built-in check for identification weakness, in the form of unbounded estimated confidence sets. Our results underscore that the choice of risk factors and models specification is a considerable challenge. It is designed to meet that challenge from an assessment of the model's specification together with the identification of such models by the data. This paper contributes to the literature by documenting the importance of this task in an international context where there are multiple sources of risk and heterogeneity across countries, time periods and dependent variables.

Based on our results, we conclude that for international asset pricing models less is more. Indeed, in the context of the empirical factor models, fewer factors with domestic sources of risk are preferable for identification, a result that might contradict the standard GRS inference. Alternatively, the international Black CAPM is well-identified and well-suited to test for financial integration. This international asset pricing model proposed by Mittoo (1992) with few benchmarks is a more promising avenue in order to test financial integration than factor-prolific alternatives.

Our joint method for model fit and identification suggests caution when relying solely on the significance of the GRS statistic to assess model fit. In our sample, domestic models had more significant GRS statistics than global models, but global models revealed more identification problems, especially when the GRS statistic was not significant. Thus, our recommendation is to look jointly at the identification of the estimated parameters as well as the GRS statistic before concluding that a model is well-specified.

Our test of financial integration is based on Black ICAPM. Our results show that the domestic sources of risk are often priced in the data. As for financial integration, it is time varying showing no trend toward integration when using the identification-robust Hotelling statistic. In terms of practical implications, the time varying and country specific nature of financial integration in this paper confirms the results of Bekeart and Harvey (1995). It also suggests that portfolio diversification and cost of capital benefits can be achieved across developed countries. However, the potential for international diversification and international hedging strategies via a specific market should be updated frequently.

Finally, the econometric approach developed in this paper has direct practical implications for developing, estimating and evaluating asset pricing models. In this literature, there is little guidance as to how many factors should be considered (e.g. Harvey et al., 2015) and researchers often add more factors in their model to avoid missing variable biases. This paper illustrates the potentially serious pitfalls of this approach in an international context where the possible sources of risk are multiplied.

Technical Appendix – Derivations of statistics

The inverted Hotelling

This Appendix summarizes results from Beaulieu et al. (2015) as they apply to the problem of inverting (18). Let the general model:

$$Y = XB + U, \quad U = WJ' \Longleftrightarrow Y_t = B'X_t + U_t, \quad t = 1, \ldots, T$$

where Y is a $T \times n$ matrix of observations on n dependent variables, X is a $T \times k$ full-column rank matrix of regressors, Y_t' and X_t' are, respectively, the tth row of Y and X so that Y_t and X_t provide the tth observation on the dependant variables and regressors, J is unknown, non-singular, U is a $T \times n$ matrix of random errors, U_t' is the tth row of U. The disturbances are i.i.d. normal with invertible contemporaneous correlation matrix. Results hold exactly imposing the multivariate normal distribution. The F-approximations used have also been shown to perform well relaxing normality.

The unrestricted Least Squares (LS) estimators are:

$$\hat{B} = (X'X)^{-1}X'Y, \quad \hat{S} = \hat{U}'\hat{U}, \quad \hat{U} = Y - X\hat{B}. \tag{A.1}$$

Let $X = [\iota_T \ \boldsymbol{X}]$ where $\boldsymbol{X}$ is $T \times q$. We refer to a $1-\alpha$ level CS for a parameter as $CS_\alpha(.)$. Partition B as follows:

$$B = \begin{bmatrix} a' \\ b \end{bmatrix}, \quad a = (a_1, \ldots, a_n)' \tag{A.2}$$

where a is the vector of n intercepts, and b is $q \times n$. Conformably partition $\hat{B}$ as follows

$$\hat{B} = \begin{bmatrix} a' \\ \hat{b} \end{bmatrix}. \tag{A.3}$$

We also partition $(X'X)^{-1}$ where x^{11} is a scalar, $x^{21} = x^{12'}$ is $q \times 1$ and x^{22} is $q \times q$:

$$(X'X)^{-1} = \begin{bmatrix} x^{11} & x^{12} \\ x^{21} & x^{22} \end{bmatrix} \tag{A.4}$$

Assume (for exposition purposes only) that one would like to solve (19), that is, to invert the Hotelling statistic in (18) numerically. One can sweep, in turn, choices for θ over a meaningful grid and for each choice considered denoted θ_0^c, where c accounts for the specific choice, compute the relevant test statistic, $\Lambda(\theta_0^c)$ and its associated p-value (using (6)). Accept θ_0^c if the statistic is not significant at the considered level, and reject otherwise. The θ_0^c vectors for which the p-value are greater than the level α collected together constitute the identification-robust confidence region with level $1-\alpha$.

From there on, a projection-based confidence set can be obtained for any function $g(\theta)$ by minimizing and maximizing the function $g(\theta)$ over the θ values included in the joint confidence region. Each component of θ can be defined as a linear combination of θ, of the form $g(\theta) = \omega'\theta$, where ω is a conformable selection vector (consisting of zeros and ones); for example, the zero-beta rate $\theta_1 = (1, 0, \ldots, 0)\theta$. Then each linear combination is maximized and minimized over θ such that $\Lambda(\theta) < F_\alpha$ (not rejected). Since $\min_\theta \Lambda(\theta) \geqslant F_\alpha \Longleftrightarrow \Lambda(\theta) \geqslant F_\alpha \ \forall\theta$, then referring $\min_\theta \Lambda(\theta)$ to F_α provides an identification-robust specification test.

The above is computationally intensive if a numerical solution is required. Conveniently, in our case, *there is no need for a numerical solution*. The analytical solution for (19) proceeds as follows. Setting $\tau_n = T - k - n + 1$, reformulate (19) as

$$\theta' A_{22}\theta + 2A_{12}\theta + A_{11} \leqslant 0 \tag{A.5}$$

$$A_{11} = \hat{a}'\hat{S}^{-1}\hat{a} - (F_\alpha(n/\tau_n))x^{11}, \ A_{12} = A'_{21} = \hat{a}'\hat{S}^{-1}\hat{b}' - (F_\alpha(n/\tau_n))x^{12}$$

$$A_{22} = \hat{b}\hat{S}^{-1}\hat{b}' - (F_\alpha(n/\tau_n))x^{22}. \quad \text{Let } \tilde{A} = -A_{22}^{-1}A'_{12}, \ \tilde{D} = A_{12}A_{22}^{-1}A_{12} - A_{11}.$$

If all the eigenvalues of A_{22} are positive then:

$$CS_\alpha(\omega'\theta) = \left[\omega'\tilde{A} - \sqrt{\tilde{D}\left(\omega' A_{22}^{-1}\omega\right)}, \omega'\tilde{A} + \sqrt{\tilde{D}\left(\omega' A_{22}^{-1}\omega\right)}\right], \quad \textit{if } \tilde{D} \geqslant 0 \tag{A.6}$$

$$CS_\alpha(\omega'\theta) = \varnothing, \quad \textit{if } \tilde{D} \geqslant 0. \tag{A.7}$$

If A_{22} is non-singular and has one negative eigenvalue then:
(i) if $\omega' A_{22}^{-1}\omega < 0$ and $\tilde{D} < 0$:

$$CS_\alpha(\omega'\theta) = \left] -\infty, \omega'\tilde{A} - \sqrt{\tilde{D}\left(\omega' A_{22}^{-1}\omega\right)}\right] \cup \left[\omega'\tilde{A} + \sqrt{\tilde{D}\left(\omega' A_{22}^{-1}\omega\right)}, +\infty\right[; \tag{A.8}$$

$$\text{(ii) if } \omega' A_{22}^{-1}\omega > 0 \text{ or if } \omega' A_{22}^{-1}\omega \leqslant 0 \text{ and } \tilde{D} \geqslant 0 \text{ then}: \ CS_\alpha(\omega'\theta) = \mathbb{R}; \tag{A.9}$$

$$\text{(iii) if } \omega' A_{22}^{-1}\omega = 0 \text{ and } \tilde{D} < 0 \text{ then}: \ CS_\alpha(\omega'\theta) = \mathbb{R} \setminus \{\omega'\tilde{A}\}. \tag{A.10}$$

The projection is given by (A.10) if A_{22} is non-singular and has at least two negative eigenvalues.

Information matrix based standard errors

The formulas for the regular standard errors of the estimates for the model parameters can be obtained from the information matrix [relevant to θ and b] which takes the form

$$\ddot{I}(\theta, b) = \begin{bmatrix} \ddot{I}_{11} & \ddot{I}_{12} \\ \ddot{I}_{21} & \ddot{I}_{22} \end{bmatrix}, \quad \ddot{I}^{-1}(\theta, b) = \begin{bmatrix} \ddot{I}^{11} & \ddot{I}^{12} \\ \ddot{I}^{21} & \ddot{I}^{22} \end{bmatrix} \tag{A.11}$$

$$\ddot{I}^{11} = \left(Tb\Upsilon^{-1}b'\right)^{-1}\left(1 - T(\bar{\boldsymbol{X}} - \theta)' Q_{\boldsymbol{X}}^{-1}(\theta)(\bar{\boldsymbol{X}} - \theta)\right)^{-1}; \quad \ddot{I}^{21} = \left(\left[-Q_{\boldsymbol{X}}^{-1}(\theta)(\bar{\boldsymbol{X}} - \theta)\right] \otimes b'\right)\ddot{I}^{11}$$

$$\ddot{I}^{22} = \left[\frac{Q_{\boldsymbol{X}}^{-1}(\theta)}{T} \otimes \Upsilon\right] + \left(\left[Q_{\boldsymbol{X}}^{-1}(\theta)(\bar{\boldsymbol{X}} - \theta)\right] \otimes b'\right)\ddot{I}^{11}\left(\left[(X - \theta)' Q_{\boldsymbol{X}}^{-1}(\theta)\right] \otimes b\right) \tag{A.12}$$

where Υ is the variance covariance of the model disturbance and $\bar{\boldsymbol{X}}$ is the $q \times 1$ vector of the (time series) column means of $\boldsymbol{X}$ and $Q_{\boldsymbol{X}}(\theta) = (\mathbf{X} - i_T\theta')'(\mathbf{X} - i_T\theta')/T$. Variance/covariance matrices for the estimators of b and θ can thus be obtained analytically using the latter expressions. The delta-method can be applied to derive the variance/covariance matrix denoted $[\ddot{A}]$ of the (constrained) estimate of the intercept: $\ddot{A} = \ddot{a}'\ddot{I}^{-1}(\theta, b)\ddot{a}, \ddot{a} = \begin{bmatrix} b \\ (\theta \otimes I_n) \end{bmatrix}$.

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