International stock market cointegration under the risk-neutral measure

Marie-Hélène Gagnon a,*, Gabriel J. Power b, Dominique Toupin c

a Université Laval, CIRPÉE, Canada
b Université Laval, CIRPÉE, CRIB, Canada
c Université Laval, Canada

A R T I C L E   I N F O

Article history:
Received 25 July 2016
Accepted 8 August 2016
Available online 10 August 2016

Keywords:
Financial market cointegration
Financial integration
Option-implied distribution

A B S T R A C T

This paper investigates international cointegration and financial integration among equity market indexes using index option data, providing an ex-ante analysis through investor anticipations. Daily time series of risk-neutral variance, skewness, and kurtosis are constructed for five major indexes for three sub-periods between 2003 and 2013. Fractionally cointegrated VAR models are estimated at the international level, accounting for persistence in risk-neutral moments. Our results show that there exist international equilibria in risk-neutral moments defined by several cointegrating vectors. During the 2007–2009 global crisis period, these equilibria are characterized by an increase in persistence and in the speeds of adjustment. Moreover, for risk-neutral variance and skewness, all markets are included in the equilibria and none are weakly exogenous. Outside the global crisis period, the cointegration relationship is more fragmented, especially for higher-order moments. In particular, crash and tail risks are segmented during the European debt crisis.

1. Introduction and motivation

1.1. Introduction

Common stochastic trends in international financial markets and financial integration have important implications for international diversification. In fact, gains to diversifying internationally are small if stock indexes share a common stochastic trend (Kasa, 1992), if indexes are financially integrated and systematic risk is priced similarly in several markets (e.g. Mittoo, 1992; Berger & Pozzi, 2013), or if there are volatility spillovers across markets (Baele, 2005; Jiang, Konstantinidi, & Skiadopoulos, 2012; Karolyi, 1995). In this literature, several authors have looked at financial crises. For instance, Eurozone sovereign debt contagion has been documented recently, with time-varying effects (Suh, 2015) and sizable joint tail risks (Lucas, Schwaab, & Zhang, in press). Mollah, Quoreshi, and Zafirov (2016) have shown that contagion has spread from US markets to international markets during the global financial crisis and the Eurozone crisis.

In this paper, rather than use historical data, we propose a novel approach using option data to investigate the common trend in risks affecting five major indexes. As option-implied data on important stock indexes are very rich, this paper provides forward-looking measures of financial cointegration and financial integration across different dimensions of risk. This paper therefore fills a gap in the literature by combining two large and active literatures, namely on the time series analysis of market cointegration, and on the information content of risk-neutral distributions. We study how shocks spread internationally under the risk-neutral measure, which better reflects the impact of disturbances (e.g., Birru & Figlewski, 2012; Konstantinidi, Skiadopoulos, & Tzagkaraki, 2008). We obtain and analyze at a daily frequency the variance, skewness, and kurtosis of the risk-neutral distribution obtained from international equity index options. Following model estimation, we perform statistical tests of cointegration, exclusion (defined as financial integration) and weak exogeneity that highlight how and to what extent anticipations spread internationally.

Our results shed new light on financial cointegration and reveal the existence of an equilibrium relationship in international anticipations, in addition to the one reported in the literature for historical returns. Our results also show that risk-neutral variance is more integrated, with greater international linkages, than are risk-neutral skewness and kurtosis. Our evidence regarding higher-order moments allows us to further document the issue and reveals that anticipations are not as integrated as ex-post returns and that fear of crashes and tail risk are locally driven. In particular, the DAX and CAC indexes are excluded.
from the higher-order moment equilibria during the sovereign debt crisis. For all moments, persistence, speed of adjustment and financial integration increase during the 2007-2009 global crisis.

1.2. Review of the literature: the relevance of risk-neutral moments in cointegration analysis

This paper contributes to the literature on cointegration analysis, market integration, and volatility spillovers. Since Kasa (1992), who documented a common stochastic trend in the quarterly equity index prices of five developed countries, cointegration analysis has been used to assess stock market interdependence (Bessler & Yang, 2003; Darrat & Zhong, 2005). There is, in particular, a growing recent literature on the process of integration in European markets (Mylonidis & Kollias, 2010; Kim, Moshirian, & Wu, 2005; Demian, 2011; Syriopoulous, 2007). For example, despite the high correlation between market indexes, Lucey and Muckley (2011) have found evidence of valuable long-term diversification potential in European markets suggesting that other metrics of risk could be relevant. Related to our contribution based on risk-neutral variance, volatility spillovers also provide evidence of financial integration (e.g., Bekker, Harvey, & Ng, 2005; Karolyi, 1995). Volatility spillovers can be driven by contagion (King & Wadhwani, 1990) and during the Eurozone crisis of 2010–2011, there was strong evidence of spillovers (Kohonem, 2013).

Nearly all studies of cointegration or volatility spillovers are, however, performed using ex-post realizations of prices or returns. Much less evidence has been uncovered to describe how forward-looking investor sentiment such as anticipations, fears, or the perception of crash and tail risk in financial markets might spill over across international markets ex-ante. This gap in the literature motivates looking at risk-neutral distributions implied from option data. Indeed, option data are informative to investors, especially in times leading to a crisis (Birru & Figlewski, 2012). For instance, incorporating option market information improves downside risk management (Brownlee, Engle, & Kelly, 2011) and portfolio selection (DeMiguel, Pfyakha, Uppal, & Vikov, 2013; Kempf, Korn, & Sæning, 2015; Kostakis, Panigirtzoglu, & Skadiopoulos, 2011). Risk managers and policymakers are concerned with higher-order moments of return distributions. It is well understood in the literature that it is challenging to recover moments of the distribution using historical data, and that moments can be estimated more reliably using option data (Bali, Cakici, & Chabi-Yo, 2011; Birru & Figlewski, 2012; Conrad, Dittmar, & Ghyssels, 2013; DeMiguel et al., 2013). Using these methods, risk-neutral moments have been shown to forecast realized returns and to have cross-sectional explanatory power (Bali & Murray, 2013; Chang, Christoffersen, Jacobs, & Vainberg, 2012; Conrad et al., 2013; Duan & Wei, 2009; Bollerslev, Osterrieder, Sizova, & Tauchen, 2013; Neumann & Skadiopoulos, 2013). Yet a comprehensive study of international linkages at the level of anticipations seems lacking.

In addition, our paper builds on a strand of the literature which suggests that risk-neutral moments in particular are related to important risk metrics in financial markets. Bakshi, Kapadia, and Madan (2003) find that when jump risk is priced in the markets, it is reflected in both the third and fourth risk-neutral moments. Duan and Wei (2009) and Denis and Mayhew (2002) find that higher-order risk-neutral moments are driven by systematic risk. Risk-neutral moments also have a meaningful economic interpretation in terms of interdependence across borders. Risk-neutral variance is associated with market expectations of near-term volatility. The VIX in the U.S. is considered to be an investor “fear gauge” (Whaley, 2000; Malz, 2013; Panigirtzoglu & Skadiopoulos, 2004). Higher-order risk-neutral moments also have an economic interpretation in terms of internationally-linked risk. Risk-neutral skewness represents the risk of negative asymmetric returns, and has been related to “crash-phobia” risk (see e.g. Jackwerth, 2000; Rubinstein, 1994). Risk-neutral kurtosis relates to tail risk or the risk associated with extreme observations under the risk-neutral measure. Both risk-neutral skewness and risk-neutral kurtosis are also related to jump risk (Pan, 2002).

1.3. Contributions to the literature

Based on this literature, we aim to measure the interdependence of investor anticipations internationally, using index option market data. Studying international time series of risk-neutral moments therefore provides a novel ex-ante measure of market cointegration and integration. Moreover, given the relevance of risk-neutral moments, this paper is distinctive for studying moments while most of the existing studies on this subject have been conducted on prices or returns.

Thus, we aim to further clarify the question of financial cointegration by disaggregating the problem into different dimensions of risk captured by variance, skewness, and kurtosis. Investigating individual moments allows us to identify which characteristic of the distribution drives the cointegrating relationship. Intuitively, each moment relates to a different type of risk that a manager might want to hedge, or alternatively get exposure to. Our contributions to the financial literature have practical implications in terms of hedging and diversification. Our results show a strong and binding equilibrium during the crisis, which suggests a lower potential for diversification and a higher degree of financial integration. However, the international equilibria for higher-order moments are more fragmented than for risk-neutral variance. These findings enrich the literature by showing there are underestimated potential benefits of international diversification for anticipations of crash risk and tail risk, but also that during the crisis period these diversification benefits are more limited.

2. Data

First, the methodology to recover the daily time series of risk-neutral moments used in the analysis is presented in this section, in a sequential order. The approach is based on Birru and Figlewski (2012).

a) Raw options data for each day and descriptive statistics

Index options data are extracted from the OptionMetrics Ivy DB database for the United States (S&P 500) and its European version for Germany (DAX), Great Britain (FTSE), France (CAC40), and Switzerland (SMI). The chosen indexes are relevant for our purposes and comparable in terms of risk, as they represent subsets of large and liquid stocks in each country. As Germany and France are often seen as the economic locomotives of the Euro currency zone, we should expect investor anticipations to be integrated across the two markets, but whether it is the case remains an empirical question. The British market and the London exchange are important to international markets, allowing us to study changes in anticipations of a large open economy outside the Eurozone. Finally, the introduction of Switzerland in the analysis provides evidence regarding smaller open economies that are outside the Eurozone, but which are deeply affected by the international flow of funds.

Table 1 presents descriptive statistics associated with the options for each index. The selected option markets are comparable in terms of the number of options traded on average each day, and in terms of implied volatility and the number of strikes per day. Given the similarities reported at the market level, the resulting risk-neutral distributions show exhibit similar levels of precisions.

Mylonidis and Kollias (2010) describe possible changes in the long-run relationship of the Eurozone market integration. To account for this possibility and to further document the differences in persistence and equilibrium relationships during crises, we divide our sample in three sub-periods: the pre-crisis period (2003/04/14 to 2007/06/21), the crisis period (2007/06/22 to 2009/06/30) and the post crisis period (2009/07/01 to 2013/12/31). The date June 22, 2007 which begins the second sub-period is chosen to reflect when it became clear to financial markets that Bear Sterns’ problems were significant (e.g., the necessary $3.2 billion secured loan) and it was considered then “an early sign of problems in the mortgage market” (Financial Times, March 22, 2008).
Because we are interested in expectations, this date appears more conservative than a later date such as the actual demise of Bear Sterns. For the end date of the crisis, we use again a conservative approach in order to make sure we isolate the crisis period. We choose July 1, 2009 to begin the third sub-period because it marks the end of the Great Recession as determined by the NBER Business Cycle Dating Committee.

b) Implied volatility surface in strike/maturity space

To construct our constant-maturity (one-month) time series, we need to fill in the implied volatility (IV) surface created from the available maturities and strikes at a certain date. Following the existing literature, we start by excluding data expiring in five days or less or having implied volatility that is negative or above 100%. The index implied volatilities are recovered directly from OptionMetrics. Cubic spline interpolation is then used to fill in this surface.

c) Option prices for constant maturities (one-month) with arbitrage-free correction

From this surface, we can obtain option prices, and we correct any arbitrage possibilities created by the interpolation procedure following the methodology in Ait-Sahalia and Duarte (2003).

d) Empirical risk-neutral distribution for available strikes at a one-month maturity

To ensure continuity in the price function, a four-degree smoothing spline is applied on our arbitrage-free IVs as suggested by Birru and Figlewski (2012). The Black-Scholes formula is used to convert back the smoothed IV curve to a call option price function, and the empirical risk-neutral distribution function $f(K)$ is obtained from Breeden and Litzenberger (1978). This function can be recovered from the second derivative of call prices.

e) Tail completion of the empirical risk-neutral distribution

Each day, options with sufficiently high and low strike prices are not always available to completely map the tails of the risk-neutral distribution. Several papers rely on giving points beyond the data either a zero weight (e.g., Bliss & Panigirtzoglou, 2004; DeMiguel et al., 2013). Given that our objective is to evaluate financial integration in higher-order risk-neutral moments, however, these approaches have the inconvenience of imposing a strong assumption regarding dispersion on the market’s anticipations as measured by the tails. We circumvent these issues by adopting the Generalized Extreme Value distributional approach (GEV). GEV has been used in numerous applications in finance (see e.g., Tsay, 2010 for details). In particular, Birru and Figlewski (2012) have used this approach to complete the risk-neutral distribution. The general intuition for the GEV approach is based on the Fisher-Tippett theorem, and is similar to the way the Central Limit Theorem makes the Normal a natural assumption regarding dispersion on the sample average from an unknown distribution. The GEV cumulative distribution function is given by:

$$F(K) = \exp \left[ - \left( 1 + \frac{K - \mu}{\sigma} \right)^{-\frac{1}{\xi}} \right]$$

(1)

For each tail, the three parameters of the GEV distribution are fitted so that the total probability in the fitted tail equals the missing total probability in this tail of the empirical risk-neutral distribution. It must also connect with the empirical RND at the 2nd and 5th or 95th and 98th percentiles. The objective function is to minimize the sum of the
squared distances between the empirical RND and the GEV distribution on the domain between each pair of connection points.

f) Computation of moments from the completed RND for a given day

The risk-neutral moments can then be computed directly from the completed distribution. The \(i\)th central moments, \(M_n\), around the mean of distribution \(c\) are given by the usual analytic formulae:

\[
M_n = \int_{-\infty}^{\infty} (x-c)^n f(x) \, dx
\]

\[
Var = \frac{M_2}{\mu^2}
\]

\[
Skew = \frac{M_3}{M_2^{3/2}}
\]

\[
Kurt = \frac{M_4}{M_2^{2}}
\]

(2)

Table 2 reports the descriptive statistics for the risk-neutral moments computed for each stock market index with a one-month constant maturity. Figs. 1, 2, and 3 present the time series dynamics for each moment, respectively, and for all indexes. The figures show that the three moments show signs of persistence, especially in the case of variance. This observation is underscored by the high autocorrelation coefficients reported in Table 2. In terms of how the series move together, a common trend emerges among the risk-neutral moments, especially for the risk-neutral skewness.

For risk-neutral skewness and kurtosis, the European market (DAX, CAC and SMl) seem to have a similar dynamic, while the US and UK markets are similar.

3. Methodology

3.1. Fractional cointegration model

Our objective is to study the interdependence of forward-looking investor sentiment internationally. Given the similarities in each given moment across several indexes, we estimate international models jointly across markets for each given risk-neutral moment. We argue that this literature provides a well-defined setting for testing the cointegrating relation as described above. The model is estimated on risk-neutral skewness and kurtosis, the European markets (DAX, CAC and SMl) seem to have a similar dynamic, while the US and UK markets are similar.

3.2. Statistical tests and hypothesis testing

The asymptotic theory required to estimate and perform statistical tests on the FCVAR model is developed in Johansen and Nielsen (2010, 2012), and applied empirically in Jones, Nielsen, and Popiel (2014). We argue that this literature provides a well-defined setting to test for cointegration, integration, and weak exogeneity. Johansen and Nielsen (2012) show that the proposed maximum likelihood estimators of \((d, \alpha, \Gamma_1, \ldots, \Gamma_k)\) are asymptotically normal, as are the estimators of \((\beta, \rho)\) for our purposes. Thus, likelihood ratios are constructed in order to test several hypotheses:

\[
\Delta^d(X_t - \mu) = \Delta^d - b t \alpha \beta' (X_t - \mu) + \sum_{i=1}^{k} \Gamma_i \Delta^d L^i (X_t - \mu) + \epsilon_t
\]

(3)

where \(\epsilon_t\) is \(p\)-dimensional IID \((0, I)\). As in traditional cointegration systems, \(\alpha\) and \(\beta\) are \(p \times r\) matrices where \(r\) is the number of cointegration vectors. The columns of \(\beta\) represent the cointegration relations, and thus \(\beta'X_t\) represents the long-run equilibrium equivalent to the stationary combinations of the variables in the system. The fractional difference operator introducing persistence in the model is \(\Delta^d\) and the fractional lag operator is \(L^i = (1 - \Delta^d)^i\). As in Johansen (2008a) and Bollerslev et al. (2013), we set \(d = b\) in order to simplify the system with a mild assumption that there is no persistence left in the cointegrating vectors. In this context, the adjustment to the long-run dynamics is captured by \(\alpha \beta'\) while the short-run dynamics are represented by \(\Gamma_i\). The variable \(\mu\) is a level parameter that shifts each of the series by a constant in order to avoid the bias related to the starting values in the sample (Johansen and Nielsen, 2016). The mean of the stationary cointegrating relation is described as: \(\beta' \mu = -\rho\). The simplified model becomes:

\[
\Delta^d(X_t - \mu) = \Delta^d - b t \alpha \beta' (X_t - \mu) + \sum_{i=1}^{k} \Gamma_i \Delta^d L^i (X_t - \mu) + \epsilon_t
\]

(4)

In our framework, Model (4) will be estimated separately three times, once for each moment. For example, when the transmission of crash risk and asymmetry from one market to another is investigated, the model is estimated on risk-neutral skewness and \(X_t\) = (RNS_VAR, RNS_RND, RNS_FTSE, RNS_SMl, RNS_CAC). Since the parameters defining the long-run equilibrium are not uniquely identified in a cointegration model (Johansen, 2000a), the \(\beta\) matrix is normalized with respect to the US market. In what follows, we define the hypotheses tested in this paper.

3.3. Statistical tests and hypothesis testing

The asymptotic theory required to estimate and perform statistical tests on the FCVAR model is developed in Johansen and Nielsen (2010, 2012), and applied empirically in Jones, Nielsen, and Popiel (2014). We argue that this literature provides a well-defined setting to test for cointegration, integration, and weak exogeneity. Johansen and Nielsen (2012) show that the proposed maximum likelihood estimators of \((d, \alpha, \Gamma_1, \ldots, \Gamma_k)\) are asymptotically normal, as are the estimators of \((\beta, \rho)\) for our purposes. Thus, likelihood ratios are constructed in order to test several hypotheses:

Table 2: Descriptive statistics for the time series of risk-neutral moments.

<table>
<thead>
<tr>
<th>S&amp;P 500</th>
<th>CAC 40</th>
<th>DAX</th>
<th>FTSE</th>
<th>SMI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nb of obs.</td>
<td>4445</td>
<td>4445</td>
<td>4445</td>
<td>2579</td>
</tr>
<tr>
<td>Mean</td>
<td>0.05</td>
<td>-0.62</td>
<td>5.62</td>
<td>0.06</td>
</tr>
<tr>
<td>Median</td>
<td>0.04</td>
<td>-0.60</td>
<td>5.41</td>
<td>0.04</td>
</tr>
<tr>
<td>Max</td>
<td>0.47</td>
<td>1.44</td>
<td>15.86</td>
<td>1.00</td>
</tr>
<tr>
<td>Min</td>
<td>0.01</td>
<td>-1.56</td>
<td>2.00</td>
<td>0.01</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.04</td>
<td>0.38</td>
<td>1.27</td>
<td>0.06</td>
</tr>
<tr>
<td>(\rho_1)</td>
<td>0.92</td>
<td>0.72</td>
<td>0.41</td>
<td>0.81</td>
</tr>
<tr>
<td>PPerron</td>
<td>-4.5*</td>
<td>-9.9*</td>
<td>-4.3*</td>
<td>-8.94*</td>
</tr>
</tbody>
</table>

Notes: The descriptive statistics are reported for the whole sample period for both assets for a one month maturity. The risk-neutral moments are: R.N.V: risk neutral variance, R.N.S: risk-neutral skewness and R.N.K: risk-neutral kurtosis. \(\rho_1\) represents the first-order autocorrelation coefficient for each series.

* Denotes a rejection of the unit root null hypothesis using the Phillips-Perron statistic with 10 lags.
a) Rank tests: cointegration test

First, a series of sequential rank tests is performed in order to identify the number of cointegrating vectors and thus, the number of stationary cointegrating relations. The hypotheses tested in the rank tests are: $H_0$: \( \text{rank} = r \) against the alternative $H_A$: \( \text{rank} = p \) [see Johansen, 1995]. The LR tests are performed for \( r = 0, 1, \ldots \) until a non-rejection is obtained.

The LR statistics used are provided in Johansen and Nielsen (2012) and the associated p-values are provided by MacKinnon and Nielsen (2014). When the rank differs from zero, we can conclude that there exists an international long-run equilibrium in the determination of a given risk-neutral moment. It would suggest that anticipations about future stock market outcomes are related together in the long run.

Fig. 1. Risk neutral volatility for the international market indexes.
b) Test on the persistence parameter, $d$.

The objective is to investigate whether the fractional integration specification is more appropriate than the traditional cointegration model for unit root systems. The hypothesis tested is:

$$H_0 : d = 1 \text{ against } H_A : d \neq 1.$$ 

The rejection of the null hypothesis implies that a FCVAR model is more appropriate than a CVAR model.

c) Tests of exclusion from the cointegrating relationship: financial integration
Testing for the presence of a cointegrating relationship via a rank test does not necessarily imply financial market integration (Kasa, 1992; Lence & Falk, 2005). Thus, we specify an exclusion hypothesis in the cointegrating relationship for which the rejection of the null is interpreted as a test of financial integration. We test whether a given market is a part of the cointegrating relationship and is thus included in an existing long-run equilibrium in risk-neutral moments. Hypotheses on $\beta$ can be formulated in the general form: $\beta = H\varphi$ where the restrictions are in the $H$ matrix of dimension $p \times s$ and $\varphi$ is an $s \times r$ matrix of free parameters (see Jones, Nielsen, & Popiel, 2014). The absence of a given country in the cointegrating relationship is tested for every country in the sample by imposing a zero restriction on the associated coefficients in the $H$ matrix. In terms of financial integration, the non-rejection of such a hypothesis would imply that

![Graphs showing risk neutral kurtosis for international market indexes](image-url)

**Fig. 3.** Risk neutral kurtosis for the international market indexes.
Table 3
Univariate estimates of fractional integration parameter (d).

<table>
<thead>
<tr>
<th>Maturity/Index</th>
<th>Risk-neutral log variance</th>
<th>Risk-neutral skewness</th>
<th>Risk-neutral kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SPX</td>
<td>DAX</td>
<td>SMI</td>
</tr>
<tr>
<td>Full sample</td>
<td>0.459</td>
<td>0.408</td>
<td>0.445</td>
</tr>
<tr>
<td>Pre-crisis</td>
<td>0.421</td>
<td>0.405</td>
<td>0.387</td>
</tr>
<tr>
<td>Crisis</td>
<td>0.554</td>
<td>0.532</td>
<td>0.607</td>
</tr>
<tr>
<td>Post-crisis</td>
<td>0.520</td>
<td>0.526</td>
<td>0.474</td>
</tr>
</tbody>
</table>

Notes: The univariate estimates of d are computed using individual time series for one risk-moment and one index at a time. Corresponding dates and numbers of observations are presented in Table 1. Log-variance is used rather than variance.

4. Results

4.1. Univariate results

To motivate a fractionally cointegrated model, we first discuss univariate results regarding long memory. Table 3 shows the univariate estimates of the fractional integration or long memory parameter d. The estimates are consistent with the joint estimates presented later. To our knowledge, it is the first evidence presented in the literature regarding the persistence of higher-order moments of European indexes. For risk-neutral variance, the estimates across market indexes range from 0.39 to 0.46 in the full sample, very close to the literature’s typical estimate of 0.40, indicating strong persistence but also far from a unit root. The estimates range from 0.38 to 0.42 in the pre-crisis period, increase during the crisis period–ranging from 0.53 to 0.61–and finally decrease in the post-crisis period to a range of 0.46 to 0.57, remaining higher than they were before the crisis. For risk-neutral skewness, full sample estimates are very similar across markets, in the range of 0.30 to 0.34. Pre-crisis, the estimates vary from 0.27 to 0.34. As is the case with variance, they increase during the crisis, ranging from 0.51 to 0.57, before falling post-crisis to values from 0.27 to 0.32. Finally, for risk-neutral kurtosis, the estimates over the full sample are the lowest of the three moments, ranging from 0.21 to 0.25. Pre-crisis, they range from 0.23 to 0.29, before increasing during the crisis period to a range from 0.30 to 0.38. They fall post-crisis to a range of 0.15 to 0.32.

4.2. Multivariate results

4.2.1. Risk-neutral variance

Table 4 presents the estimation results for the FCVAR model applied to risk-neutral variance for the three sub-periods studied in this paper. The models are well-specified. The null of standard cointegration $H_0$: $d = 1$ is rejected in every sub-period, suggesting that a fractional cointegration model is appropriate. This finding further documents, with international evidence, the relevance of long memory models for the analysis of risk-neutral moments, corroborating the evidence on the VIX and realized variance presented in Bollerslev et al. (2013). The models also contain enough lags, as the Breusch-Godfrey tests of serial autocorrelation of the residuals fail to reject the absence of autocorrelation.

In terms of model estimation, the joint estimate of the long memory parameter across five markets is respectively $d = 0.42, 0.56$ and 0.52 for the three sub-periods. The pre-crisis estimate is in line with the existing literature, which estimates the persistence of variance at around 0.4 (Andersen, Bollerslev, Diebold, & Ebens, 2001; Christensen & Nielsen, 2007; Bollerslev et al., 2013). The results also document highly persistent risk-neutral variance during the crisis, consistent with previous research that has found increasing long memory in realized volatility during periods of crisis (e.g., Asai, McAleer, & Medeiros, 2012). This result confirms internationally, under the risk-neutral measure, well-known stylized facts under the physical measure such as the CIR leverage effect during periods of crisis and volatility clustering being greater during recessions (Hamilton & Lin, 1996). We also note that persistence in variance remains elevated in the post-crisis period, though lower than during the crisis period. Given that our sample contains many European countries, this result can be partly linked to the European sovereign debt crisis. These results, regarding persistence in risk-neutral variance during and after the crisis but rejecting a unit root, underscore the relevance of the proposed fractional cointegration model, which accounts for persistence without losing information due to differencing. Indeed, given long memory, neither the stationary VAR nor the standard cointegration model would be appropriate.

Rank tests document the presence of a long-term equilibrium and also evidence of its variation over time. In fact, the number of cointegrating vectors goes from three before the crisis to two during and after the crisis. A strand of the existing literature constrains the number of cointegration vectors to one (e.g., Mylonidis & Kollias, 2010). Our rank tests show that the specification of a single cointegrating vector is never retained by our model selection methodology. Our results show that this constraint is restrictive under the risk-neutral measure.

Table 4 also presents p-values for the hypotheses on $\beta$ and $\alpha$. We follow Jones, Nielsen, and Popiel (2014) and use the 10% level of significance to reject a given null hypothesis. In terms of the components of the cointegrating relation, the results presented for $\beta$ confirm that all the considered indexes enter the cointegrating relation during the crisis. Moreover, tests of weak exogeneity suggest that investor anticipations in all the countries adjust when a shock occurs. These findings suggest that investor anticipations crystallized in the risk-neutral variance are integrated internationally in times of crisis. Our finding therefore strengthens the existing evidence regarding the ex-post existence of contagion of market returns during a crisis (Bae, Karolyi, & Stulz, 2007; Bollerslev et al., 2013).
For example, during the crisis, the error correction term. When there are two cointegrating vectors, the exogenous. The Swiss and FTSE index risk-neutral variances are weakly occurring in the anticipations for the other countries. After the crisis, that anticipations in those two markets do not adjust to shifts occurring outside the crisis period, investor anticipations regarding British markets are not driven by a single global equilibrium but rather that the FTSE index is excluded from the cointegrating relationship before and after the crisis period. This result suggests that the anticipation relationship directly from market anticipations. We (2003) and also presents ex-ante evidence of integration and contagion through market anticipations. However, the equilibrium is different outside the crisis period. We find that the FTSE index is excluded from the cointegration relationship before and after the crisis period. This result suggests that outside the crisis period, investor anticipations regarding British markets are not driven by a single global equilibrium but rather partly driven by domestic factors. In the pre-crisis period, the S&P and FTSE indexes are found to be weakly exogenous, which implies that anticipations in those two markets do not adjust to shifts occurring in the anticipations for the other countries. After the crisis, the Swiss and FTSE index risk-neutral variances are weakly exogenous.

Table 5 presents the adjustments following a shock in each index’s risk-neutral variance. As pointed out in Kasa (1992), the economic relevance of the cointegration relationship and resulting cointegration also depends on the speed of adjustment implied by the model. The outcome of the adjustment mechanism of the system following a particular shock is computed following Jones, Nielsen, and Popiel (2014). In fact, the adjustment process cannot be obtained directly from $\alpha$ when there is more than one cointegrating relationship. Let $t_1 = \beta(X_t - \bar{\mu})$ be the long-run equilibrium. Thus, $t_1$ is the error correction term. When there are two cointegrating vectors, the adjustment dynamics describing the response of a given variable to a specific 1% increase in the risk of one index are obtained by adding the effect from each cointegrating vector. For example, during the crisis, the international system of risk-neutral variances is described by two cointegration vectors:

$$SPX_t = 6.32 \times FTSE_t + 2.808 \times DAX_t + 6.552 \times CAC_t + \upsilon_{1t}$$

$$SMI_t = -14.759 \times FTSE_t + 6.14 \times DAX_t + 18.25 \times CAC_t + \upsilon_{2t}$$

Consider as an example how to find the adjustment dynamics of the DAX index variance to a one-percent increase in the variance of the FTSE index. Holding everything else constant, an increase of 1% in FTSE will have a corresponding effect on the equilibrium variances: $\upsilon_{1t} = -6.32$ and $\upsilon_{2t} = 14.76$. Thus, the adjustment associated with the fractional difference in DAX is:

$$\alpha_{DAX} \upsilon_{1t} = -0.18 \times 6.32 + 0.38 \times 14.76 \times 0.06 = -1.51$$

The magnitudes of adjustment are larger during times of crisis. Larger values of adjustment indicate a faster return to equilibrium and a higher degree of stock market convergence (Mylonidis & Kollias, 2010). Indeed, during the crisis, shocks in one country are often met with larger responses in other countries. However, outside the crisis period, the adjustments are quite small, suggesting that in non-crisis periods, anticipations are not highly responsive to international shocks.

### 4.3 Risk-neutral skewness

Table 6 presents the results for international fractional cointegration models applied to risk-neutral skewness, often referred to as “crash-aphobia” (Rubinstein, 1994). In terms of model fit, the hypothesis of a unit root is rejected and the test on $d$ and on the residuals show that fractional cointegration models are well-specified. The estimates of...
The top part of the table reports the optimal number of lags representing the short run dynamics and the MLE estimates for the cointegrating relations between different stock index anticipations. The bottom part of the table reports the p-values for the test of exclusion and weak-exogeneity as well as the p-value of the absence of autocorrelation in the residual test with account for the last week (Q(5)). Following Jones, Nielsen, and Popiel (2014), the significance level is set to 10% for exclusion and weak exogeneity tests.

<table>
<thead>
<tr>
<th>Lags</th>
<th>Pre-crisis</th>
<th>Crisis</th>
<th>Post-crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Estimate of $d$</td>
<td>0.18 (0.03)</td>
<td>0.60 (0.05)</td>
<td>0.32 (0.04)</td>
</tr>
<tr>
<td>Cointegrating relations $β$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SPX</td>
<td>1.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>SMI</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>FTSE</td>
<td>−0.03</td>
<td>0.23</td>
<td>0.00</td>
</tr>
<tr>
<td>DAX</td>
<td>0.81</td>
<td>−0.54</td>
<td>0.00</td>
</tr>
<tr>
<td>CAC</td>
<td>−0.21</td>
<td>−0.33</td>
<td>−0.77</td>
</tr>
<tr>
<td>Adjustment matrix $α$</td>
<td>0.41</td>
<td>−0.01</td>
<td>−0.29</td>
</tr>
<tr>
<td>Ho: $d = 1$</td>
<td>$p = 0.00$</td>
<td>$p = 0.00$</td>
<td>$p = 0.00$</td>
</tr>
<tr>
<td>Ho: SPX weakly exogenous</td>
<td>$p = 0.00$</td>
<td>$p = 0.00$</td>
<td>$p = 0.00$</td>
</tr>
<tr>
<td>Ho: SMI weakly exogenous</td>
<td>$p = 0.00$</td>
<td>$p = 0.00$</td>
<td>$p = 0.00$</td>
</tr>
<tr>
<td>Ho: FTSE weakly exogenous</td>
<td>$p = 0.00$</td>
<td>$p = 0.00$</td>
<td>$p = 0.00$</td>
</tr>
<tr>
<td>Ho: DAX weakly exogenous</td>
<td>$p = 0.00$</td>
<td>$p = 0.00$</td>
<td>$p = 0.00$</td>
</tr>
<tr>
<td>Ho: CAC weakly exogenous</td>
<td>$p = 0.00$</td>
<td>$p = 0.00$</td>
<td>$p = 0.00$</td>
</tr>
<tr>
<td>Q(5)</td>
<td>$p = 0.46$</td>
<td>$p = 0.13$</td>
<td>$p = 0.69$</td>
</tr>
</tbody>
</table>

The top part of the table reports the optimal number of lags representing the short run dynamics and the MLE estimates for $β$ and $α$ as well as their associated standard error in parenthesis. The bottom part of the table reports the p-values for the test of exclusion and weak-exogeneity as well as the p-value of the absence of autocorrelation in the residual test with five lags to account for the last week (Q(5)). Following Jones, Nielsen, and Popiel (2014), the significance level is set to 10% for exclusion and weak exogeneity tests.

The rank tests show an increasing number of cointegration vectors, thus fewer common stochastic trends, during the crisis. In fact, there is only one common international source of permanent shocks to asymmetric risk anticipations during the crisis (as $r = 4$). The number of cointegrating vectors is two before and three after the crisis period, implying more than one long-run common stochastic trend outside the crisis period.

Furthermore, our results in terms of inclusion and weak exogeneity confirm that the international equilibrium in investor anticipations has a more prominent role during the crisis period. Indeed, risk-neutral skewness for every index belongs to the cointegrating relationship and no markets are weakly exogenous during the crisis. Outside the crisis period, however, our results show once more that the international equilibrium in investor anticipations is more fragmented. In particular, FTSE and CAC are not in the cointegrating relationship before the crisis, while DAX and CAC are excluded from the equilibrium after the crisis. The post-crisis exclusion from the cointegrating relationship of Germany and France can be linked to the European sovereign debt crisis, and suggests that market anticipations regarding asymmetric risk were determined regionally for the Eurozone during this period. Finally, our results outside the crisis period show that several markets are weakly exogenous. Before the crisis, SMI, FTSE, and CAC are weakly exogenous. After the crisis, SPX, FTSE, and DAX are weakly exogenous.

Table 7 presents the speed of adjustment resulting from a disequilibrium. The adjustments are larger during the crisis, but small in general. This finding implies that—although there exists a cointegrating relationship between different stock index anticipations of asymmetric risk—adjustments from any international disequilibrium are modest. In practical terms, these results suggest a smaller level of international linkages in asymmetric risk even during the crisis period. Taken together, the evidence on the exclusion of the equilibrium and the modest magnitude of the adjustments outside the crisis period suggest that the adjustments in anticipations of asymmetric risk are less affected by the international equilibrium and are more subject to local events. The findings also suggest that regional crash risk, such as perhaps the European 2010 debt crisis, could be hedged through international diversification.

### 4.4. Risk-neutral kurtosis

Tables 8 and 9 present the results for the FCVAR model applied to risk-neutral kurtosis, capturing tail risk anticipations. For the model fit, the estimates in persistence ($d$) show that that risk-neutral kurtosis is less persistent than risk-neutral variance or skewness, but still significant at around 0.2. As with the other moments, kurtosis is cointegrated internationally with two cointegrating vectors for each of the sub-periods considered.

Although there is a cointegrating relationship, the hypotheses testing on $β$, $α$ and the speed of adjustment document a more fragmented international relationship for risk-neutral kurtosis compared to the other moments. In fact, we find that the equilibrium excludes several indexes from the relationship. Before the crisis, SPX is excluded and is weakly exogenous, while FTSE is weakly exogenous. During the crisis, CAC is excluded and DAX is weakly exogenous. After the crisis, Eurozone countries (CAC and DAX) are excluded and weakly exogenous. This result confirms the assessment by investors that tail risk in the Eurozone was different, perhaps because it was more affected by regional factors related to the Eurozone. Table 9 shows that adjustments before the crisis are small, while adjustments during the crisis are larger. The larger speed of adjustment after the crisis should be interpreted with caution as many of these estimates are obtained from non-significant values of $α$. We find that taken collectively, our results show that a single
In this paper, we study international financial market cointegration using a fractionally cointegrated VAR model applied to risk-neutral variance, skewness, and kurtosis obtained from options data on five major market indexes. Studying moments allows us to differentiate international linkages in volatility, crash, and tail risks. Thus, using a well-defined statistical setting, our paper documents ex-ante cointegration, financial integration, and weak exogeneity in investor anticipations. We find that that a fractional cointegration model provides a good representation of the daily time series of arise-neutral variance, skewness, and kurtosis across five markets. Our equilibrium is characterized by several vectors of cointegration, though fewer than under the physical equilibrium after the crisis for all indexes seems less appropriate. These results document further that the Eurozone debt crisis did not spread internationally as much as did the credit freeze crisis, and that it remained largely regional (Argyrou & Kontonikas, 2012).

### 5. Discussion and conclusion

In this paper, we study international financial market cointegration using a fractionally cointegrated VAR model applied to risk-neutral variance, skewness, and kurtosis obtained from options data on five major market indexes. Studying moments allows us to differentiate international linkages in volatility, crash, and tail risks. Thus, using a well-defined statistical setting, our paper documents ex-ante cointegration, financial integration, and weak exogeneity in investor anticipations. We find that that a fractional cointegration model provides a good representation of the daily time series of risk-neutral variance, skewness, and kurtosis across five markets. Our equilibrium is characterized by several vectors of cointegration, though fewer than under the physical equilibrium after the crisis for all indexes seems less appropriate. These results document further that the Eurozone debt crisis did not spread internationally as much as did the credit freeze crisis, and that it remained largely regional (Argyrou & Kontonikas, 2012).

2 Although our analysis is not on prices but rather on risk-neutral moments, we perform a robustness exercise to control for the exchange rate and its documented persistence (see e.g. Cheung, 1993). The moneyness of each option used to recover the risk-neutral distribution is converted in US dollars using the spot exchange rate for the spot and the one-month forward exchange rate for the strike price. Thus, we control for the possibility that the expectations regarding the exchange rate are driving our models. Detailed results are omitted for brevity, but do not change the interpretation of the findings in terms of persistence or the relevance of FCVAR, as the optimal models are similar to the one reported above.

---

### Table 7

The adjustment dynamics of international risk-neutral skewness.

<table>
<thead>
<tr>
<th>Shock on SPX</th>
<th>Pre-crisis</th>
<th>Crisis</th>
<th>Post-crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPX</td>
<td>0.41</td>
<td>-0.29</td>
<td>0.20</td>
</tr>
<tr>
<td>SMI</td>
<td>0.11</td>
<td>0.06</td>
<td>0.41</td>
</tr>
<tr>
<td>FTSE</td>
<td>0.23</td>
<td>0.10</td>
<td>0.02</td>
</tr>
<tr>
<td>DAX</td>
<td>0.12</td>
<td>0.16</td>
<td>0.20</td>
</tr>
<tr>
<td>CAC</td>
<td>0.14</td>
<td>-0.77</td>
<td>0.17</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shock on SMI</th>
<th>Pre-crisis</th>
<th>Crisis</th>
<th>Post-crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPX</td>
<td>0.12</td>
<td>0.15</td>
<td>0.04</td>
</tr>
<tr>
<td>SMI</td>
<td>0.13</td>
<td>-0.22</td>
<td>-0.31</td>
</tr>
<tr>
<td>FTSE</td>
<td>0.28</td>
<td>0.37</td>
<td>0.29</td>
</tr>
<tr>
<td>DAX</td>
<td>-0.29</td>
<td>0.54</td>
<td>-0.09</td>
</tr>
<tr>
<td>CAC</td>
<td>0.24</td>
<td>0.33</td>
<td>-0.18</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shock on FTSE</th>
<th>Pre-crisis</th>
<th>Crisis</th>
<th>Post-crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPX</td>
<td>-0.01</td>
<td>-0.21</td>
<td>-0.18</td>
</tr>
<tr>
<td>SMI</td>
<td>0.00</td>
<td>0.11</td>
<td>-0.08</td>
</tr>
<tr>
<td>FTSE</td>
<td>0.00</td>
<td>-0.61</td>
<td>-0.22</td>
</tr>
<tr>
<td>DAX</td>
<td>-0.01</td>
<td>0.29</td>
<td>-0.16</td>
</tr>
<tr>
<td>CAC</td>
<td>0.00</td>
<td>0.26</td>
<td>0.04</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shock on DAX</th>
<th>Pre-crisis</th>
<th>Crisis</th>
<th>Post-crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPX</td>
<td>0.27</td>
<td>0.01</td>
<td>-0.08</td>
</tr>
<tr>
<td>SMI</td>
<td>0.02</td>
<td>-0.02</td>
<td>-0.50</td>
</tr>
<tr>
<td>FTSE</td>
<td>0.04</td>
<td>0.05</td>
<td>0.27</td>
</tr>
<tr>
<td>DAX</td>
<td>0.26</td>
<td>-0.70</td>
<td>-1.00</td>
</tr>
<tr>
<td>CAC</td>
<td>-0.02</td>
<td>0.07</td>
<td>0.16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shock on CAC</th>
<th>Pre-crisis</th>
<th>Crisis</th>
<th>Post-crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPX</td>
<td>-0.12</td>
<td>0.30</td>
<td>-0.26</td>
</tr>
<tr>
<td>SMI</td>
<td>-0.07</td>
<td>0.02</td>
<td>-0.65</td>
</tr>
<tr>
<td>FTSE</td>
<td>-0.14</td>
<td>0.20</td>
<td>0.02</td>
</tr>
<tr>
<td>DAX</td>
<td>0.07</td>
<td>-0.63</td>
<td>0.38</td>
</tr>
<tr>
<td>CAC</td>
<td>-0.11</td>
<td>-0.62</td>
<td>-0.61</td>
</tr>
</tbody>
</table>

---

### Table 8

Estimation results for the FCVAR models using risk-neutral kurtosis.

<table>
<thead>
<tr>
<th>Lags</th>
<th>Pre-crisis</th>
<th>Crisis</th>
<th>Post-crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.21</td>
<td>0.22</td>
<td>0.18</td>
</tr>
<tr>
<td>2</td>
<td>(0.03)</td>
<td>(0.05)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>3</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>4</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>5</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

The top part of the table reports the optimal number of lags representing the short-run dynamics and the MLE estimates for $\beta$ and $\alpha$ as well as their associated standard error in parenthesis. The bottom part of the table reports the $p$-values for the test of exclusion and weak exogeneity as well as the $p$-value of the absence of autocorrelation in the residual test with five lags to account for the last week (Q(5)). Following Jones, Nielsen, and Popiel (2014), the significance level is set to 10% for exclusion and weak exogeneity tests.

2 The presence of a small number of common trends is seen in the literature as evidence of strong financial integration. When looking at anticipations, we conclude there is partial financial integration, a finding similar to Bley (2009) and Lucey and Muckley (2011). All three moments display long memory. In particular, the persistence of risk-neutral variance is in line with the literature on real volatility and VIX (Bollerslev et al., 2013). The evidence of persistence of higher-order risk-neutral moments is new to the literature. Persistence and the speed of adjustment to a disequilibrium increase during the financial crisis period. The crisis period is also distinctive in our sample as our results show a greater level of financial integration, and more contagion of anticipations than before or after the 2007–2008 crisis. Finally, our analysis suggests that looking at moments provides useful new insights. Indeed, our results regarding risk-neutral variance imply strong integration both during and outside the global financial crisis. This result complements the existing literature that finds important volatility spillovers and financial integration in stock index prices (Kohonen, 2013; Kim et al., 2005; Morana & Beltratti, 2008). Our analysis extends the question to higher-order moments, providing evidence that crash and tail risk measured via option market data are not as integrated as variance. In particular, crash and tail risk equilibrium exclude France and Germany during the sovereign debt crisis. This result extends evidence of unprecedented tail risk in Europe in 2011–2012 as presented in Lucas et al. (in press), and suggests that this risk did not spread heavily to the international equilibrium. Taken collectively, our results suggest that crash and tail risks are more diversifiable internationally than variance. While this diversification potential is important outside the global crisis, it is reduced during the global meltdown.


